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THE JOURNAL OF PHILOSOPHY

WHITEHEAD CENTENNIAL ISSUE *

WHITEHEAD AS I KNEW HIM

THOSE who were teaching at Harvard when Alfred Whitehead joined us in the Fall of 1924 knew from the first that we were peculiarly fortunate. We also realized—and had intended—that our new member should be an exception to some of our customary rules—for example, the customary retirement at sixty-five. Whitehead was sixty-three when he came. And since the transplanting of his labors from England to America meant that he had ideas to develop which would require their own leeway, that time-limit fell away automatically, with President Lowell in full agreement. He was to be with us for at least five years.

As it turned out, Whitehead continued his active teaching for thirteen years, retiring at 76, and remained with us for ten years more. During most of this time he gave little impression of age: though senior to all of us, he was in all daily doings completely contemporary and a vigorous sharer in the community of thinking that constitutes a "department." One of us said of him, "Whitehead is the youngest man I know!"

Though he came to us with the aura of leadership, he was temperamentally devoid of every gesture or assumption of authority. When he spoke, we heard a voice notably equable and unassertive. But when he said "I *think* . . ." so and so, we listened to an authentic bit of thinking, coming from fresh impressions—original in the literal sense of the word, often with a ready humor and with a frequent resort to the colloquial quip. As when, in dilating on his theme of process, he remarked,

> Reality is becoming; it is passing before one—a remark too obvious to make. . . . You can't catch a moment by the scruff of the neck—*it's gone*, you know.

Or when, in swiftly disposing of the mystery of how we know minds other than our own, he declared,

> Hang it all! *Here we are.* We don't go behind that; we begin with it.

* Editor for this special issue: George L. Kline.

He was at home in the vernacular. And in part, on principle. For while it was at once apparent that Whitehead was doing the necessarily lonely work of formulating a new vision of the universe, it was also evident that he was working not from one foundation, but from two: the highly technical thought of advanced analysis, and the immediate experience of common humanity. In one of his early lectures he asserted, "I am a very naive individual"; fully aware that the purely naive don't know it, still less say it! In any case, he carefully preserved his bonds of kinship with the man of the street.

And this bond, by the way, was of the greatest use in retaining, as he did, his hold upon undergraduate students, whom he unsparingly led into rugged paths, and in a terminology already strewn with coins of his own mintage. For his speculative structure, which came to fruition during his American years, was already well advanced in its main outlines. Had this not been the case, he would have lacked the compelling motive to cross the ocean. Any impression that he began his mature philosophical work in America is far from the fact.

What was the motive for this grave decision on his part? For it was a grave decision, even though he remained within the British tradition and in a sense never left Great Britain, whether in affection or in political belonging: "in spite of all temptation . . . he remained an Englishman." It is also true that he was as little a captive of space as of time: he seemed almost as one insulated from locality. It remains true that the step was, for him and his family, momentous, and not least so on account of the growing scope of his philosophical audience on the Continent as well as in England; Bergson had given his opinion that Whitehead was the most significant philosopher writing in English. I agree with his son, North Whitehead, who has kindly answered an enquiry of mine on this point, that we seldom know the full reason for any important decision, and that my suggestion of the "wider field of philosophical reflection" open to him at Harvard as a defined topic of teaching, while to the point, was probably not the whole motive. But from my own conversations with Whitehead I have a further clue, substantiated by certain facts which, in the history of Harvard, had laid the foundation for his decision.

In Sir Charles Snow's Rede Lecture of 1959 on "The Two Cultures and the Scientific Revolution" there is pictured a cleavage in a typical British university—a cleavage in understanding between the traditional humanists and the oncoming scientific wave.

Allowing for a degree of exaggeration in this picture, it is true that the Cambridge in which Whitehead was first a student and then a don was inclined to sharp specialization. As a student there, Alfred Whitehead attended lectures on mathematics and *nothing else!* When I first heard him make this statement I was incredulous; for if I had to name a man of exemplary breadth of culture, I should think first of Whitehead. "There must have been something else," I said. "Yes," he admitted, "there was something else, but it was not by way of the academic routine. It was largely 'The Apostles.'"[1]

In the American Cambridge of the same period a quite different spirit was at work: science and the "genteel tradition" were finding common ground. William James had come into philosophy from medicine and biology, having opened a psychological laboratory in Dane Hall (perhaps the first in the land), outfitted with calves' brains, scalpels, microscopes—to the scandal, it must be confessed, of certain colleagues. But it was Josiah Royce who most effectively welded the disciplines of science into the work of philosophy and made this welding a part of the Harvard temper.

It was Royce who introduced into his lectures on metaphysics enquiries into the mathematics of infinite collections. It was he who first offered courses in symbolic logic, and began technical enquiries into scientific method. His interest in these topics was stirred by conversations with Charles Peirce and guided by unpublished lectures of that restless and fecund spirit, for whom he, with James, had solicitous regard and respect. Royce gathered around him a number of colleagues—astronomers, biologists, physicists, mathematicians—meeting usually on Sunday evenings for supper and discussion. I recall one evening when Royce opened the debate with a paper on statistical methods, later printed in *Science*. After Royce's death in 1916, this group carried on as "The Royce Club." Professor E. B. Wilson tells me that in the 1920s there were 26 members.

It now transpires that members of this Royce Club played a decisive part in paving the way for Whitehead's coming to Harvard.

[1] A group of fellow students limited to twelve at any one time, said to trace its origins to Tennyson and Hallam. Its proceedings and membership were strictly secret. It met on Saturday evenings at ten in the room of some member. An appointed member read a paper on some theme which concerned him. The remaining members drew lots, and in order of their numbers must stand each with his back to the fireplace and state without reserve precisely what he thought of the paper. "The revelations were often extraordinary. Themes were developed that remained permanent interests."

Already in 1920 the Department of Philosophy had become interested in this possibility. Professor Woods, then chairman, had written in March to President Lowell in that interest, but there were financial problems that could not then be solved. It was not until 1923 that the issue was revived. It seems that Lawrence Henderson, biologist and Royce Club member—probably during a summer visit to England—had learned of Whitehead's impending retirement from the University of London. A bit later, when he with Professors Wheeler and Wilson (also Royce Club members) were week-end guests of Henry Osborn Taylor, he mentioned the matter; and there was common agreement that Whitehead's work was far from finished and that the possibility of his coming to America should be actively explored. From that resolve to the actual invitation, there were steps involving university policy and finance—steps in which Taylor's practical intervention was decisive.

By January of 1924, Whitehead had been informed through a friend and fellow scientist, Mark Barr, that there was "a chance at Harvard." On January 13th, Whitehead wrote to this intermediary:

> . . . If the post should be offered to me, I should find the idea of going to Harvard for five years very attractive. The post might give me a welcome opportunity of developing in systematic form my ideas on Logic, the Philosophy of Science, Metaphysics, and some more general questions, half philosophical and half practical, such as Education . . . I do not feel inclined to undertake the systematic training of students in the critical study of other philosophers. . . . If however I should be working with colleagues who would undertake this side of the work, I should greatly value the opportunity of expressing in lectures and in less formal manner the philosophical ideas which have accumulated in my mind.[2]

On February 4, President Lowell sent by way of Taylor a formal offer to Whitehead, with the appended remark:

> If you cable, you can add that the professors in the Department of Philosophy are delighted with the prospect of his coming.[3]

On February 24, Whitehead accepted the post for five years, effective September, 1924.[4] I am told that when the proposal came and was first mentioned by Whitehead to his family as a concrete possibility requiring joint decision, it was with the remark, "I have long wanted to teach philosophy." Fortunately, Mrs. Whitehead was by temperament disposed to adventure.

[2] Quoted from official records with the kind permission of the President and Fellows of Harvard College.

[3] *Ibid.*

[4] *Ibid.*

In this tenuous chain of events, there is a large element of accident; there is also a substantial element of mutual fitness—let us say an unplanned convergence. Harvard had laid a foundation peculiarly favorable to Whitehead's constructive genius by developing a collegiate atmosphere in which the mutual insulation of Sir Charles Snow's "two cultures" was simply nonexistent. The two cultures were there in full vigor, definitely not alien, but duly fused and seeking joint philosophical expression. The cleavage characteristic of the "modernity" stemming from Descartes, and imperfectly mended by a series of great unifiers from Spinoza and Leibniz to our own day, was here rejected in practice and moving toward philosophical solution. To this extent, it is fair to say that we owe Whitehead's presence to this group of colleagues, and ultimately to Royce.

Whitehead himself was, of course, unaware of this connection. He came indeed to hold in high regard the "great period" of Royce and others, having the essential merit of "a group of adventure, of speculation, of search for new ideas" (MT 237).[5] But he was immediately influenced by what he already knew of the general spirit of America and its higher education. He had once visited this country. He lectured at Bryn Mawr, April 18, 1922, on "Some Principles of Physical Science." There, as his son, North Whitehead, puts it, he "fell in love with America" as realizing some of the ideals for which he had been laboring in England —a wider access of the people to the universities and especially an ample provision for the higher education of women. He knew something of what he could anticipate; and I feel confident that be continued to approve his step. In North Whitehead's words, "My father not only did not regret crossing the Atlantic. He

[5] References to Whitehead's principal works will be identified by the following abbreviations throughout this special issue of the JOURNAL:

AI	*Adventures of Ideas.* New York: The Macmillan Company, 1933.
CN	*The Concept of Nature.* Cambridge: At the University Press, 1920.
ESP	*Essays in Science and Philosophy.* New York: Philosophical Library, 1948.
MT	*Modes of Thought.* New York: The Macmillan Company, 1938.
PM	*Principia Mathematica* (with Bertrand Russell). Cambridge: At the University Press, 3 vols. (PMI, PMII, PMIII), 1910–1913.
PR	*Process and Reality.* New York: The Macmillan Company, 1929.
R	*The Principle of Relativity.* Cambridge: At the University Press, 1922.
RM	*Religion in the Making.* New York: The Macmillan Company, 1926.
SMW	*Science and the Modern World.* New York: The Macmillan Company, 1925.

thought it one of the best things he ever did; he genuinely loved this country and believed in it."[6]

No doubt, a part of its promise, in his eyes, lay in our own unfinishedness and our faith in directed change: it was this which attracted him to the work of William James and John Dewey. For in his view, "The use of philosophy is to maintain an active novelty of fundamental ideas illuminating the social system" (MT 237), an expression which by itself seems to involve a definite waiver of finality. In this sense, America could impress him as holding a greater promise, for explorative thought, than the Old World; and indeed, in a late letter to Professor Charles Hartshorne, he indicated his impression that the (next) future of philosophy was to be in America, Europe being hopelessly entangled in traditional modes of thought.

This is in accord with his thoroughly individual attitude toward the history of philosophy. No one has touched that history with more masterly assurance throughout its western extent from Plato to Weierstrass, Einstein, Niels Bohr; yet Whitehead showed equally masterful selectivity—for there were considerable stretches not relevant to his work, which he could regard (with Samuel Crothers) as "honorable points of ignorance." He felt here a definite release from the Anglo-European pressure on details of that history: "At home I should have been constantly reminded," he remarked, "that I know too little of Hegel—a man whom I have found writing such nonsense about mathematics that I am not tempted to pursue his thought in depth." He was prone in lecturing to wave a playful disclaimer of historical lore: "But you know far more about these things than I do!" Yet the play covered a serious conviction, which so far as I recall he had not formulated, but which may be stated thus: The history of philosophy is never "learned" nor learnable; for it, too, grows with that "active novelty of fundamental ideas" and has to be restated with every radical advance. In this sense, Whitehead frequently knew the history of thought far better than that history (via its exponents) knew itself.

It belonged to the situation that his topics were his own; and it turned out that during his first year with us, while the university catalogue for 1924–1925 mentioned three courses over his name: one full course for undergraduates, two "seminary" courses for graduates, each a half-year program, there was just one topic for them all: "The Philosophical Presuppositions of Science"!

[6] Letter of January 19, 1961.

Of personal recollection there is far more to relate than I can possibly compress into this note. Let me simply mention three facets of this rare experience: our joint seminar in metaphysics; Whitehead's way with students, more especially his mode of conveying criticism; the paradox in his ultimate aim.

The joint seminars. Long ago—about 1910—when I was teaching in Yale, I brought to President Hadley a rather audacious proposal: that in topics open to discussion, it is too easy for an instructor to refute a doctrine he disagrees with in the absence of its proponent. *The proponent should be there.* The students should enjoy the benefit of the living debate. Hadley agreed. But he noted the practical difficulty, for a university needing to widen its offering, of expending the energy of *two* teachers for *one* exercise.

This difficulty prevented my realizing the idea until I could find colleagues so overflowing in their own resources that they could contribute an extra hour or two per week without undue exertion. At Harvard I had the rare good fortune to find two such colleagues: Roscoe Pound in philosophy of law, Alfred Whitehead in metaphysics.

For the spring term of 1933–1934—Whitehead was then finishing his tenth year with us—we planned (for graduate students, and for ourselves) a two-headed exploration of topics of mutual interest and difficulty. The title of this experiment was "Seminary in Metaphysics" (Philosophy 20h), meeting Friday afternoons from 3:30 to 5:30. We were counting on the advantages of binocular vision.

The topics were naturally chosen from matters in which there was a divergence of views as between AW and EH; but the interest was in the enquiry—the conflict was incidental, representative of a major human problem. We were to take turns in presenting our analyses, and open the matter for discussion by the students.

The general theme was "Necessity and Contingency": what features of the world we live in are what they necessarily have to be? what features are pure matter of fact, accident, or chance? For example, are space and time, mass and energy, life and the lifeless, necessary features of every possible world? or are they factual conditions that might be otherwise? If they are necessary, it becomes the task of metaphysics to "deduce the categories" from some necessary first principle, the ideal of a rationalist philosophy. If they are contingent, we must proceed empirically, reporting what we find as traits of the existing world epoch,

subject perhaps to slow change; metaphysics then becomes a problem of description, in terms adequate and consistent, the aim of Whitehead's *Process and Reality*. I was far from being a "rationalist," but held that there were elements of necessity in our world and that the search for necessity was—for metaphysics—a necessary search.

The success of this joint enterprise obviously depended on complete frankness on both sides, and on the part of the students as well. Whitehead was most insistent on this point: in the nature of any new point of view there is required "a certain measure of ferocity." When a student modestly suggested that his point of criticism "may be superficial," Whitehead's swift word was, "Don't be polite to me." After accusing me on one occasion of having spoken from a Hegelian standpoint, he added, "Don't be afraid of exposing my ignorance before the class—say anything you like—I speak in complete ignorance of Hegel."

He was consistently unsparing in characterizing *my* views, which he condemned as "too orthodox," assuming naturally, but a bit hastily, that I stood squarely in the Bradley-Royce tradition. In one of our preparatory sessions (we used to meet for lunch on Thursday to outline the next day's discussion) he said, "You are always using words that are awfully suspect. I am always in doubt whether I ought not to burn you." On a suggestion of mine that we mean by "the Real" the *source* of things-and-events, that which *explains* what is and what occurs, he commented,

> You are too intellectual and dialectical; you have too much trust in linguistic discussion. The Real does not explain; it is the Doctrine that explains. Like the drunk who meandered from one lamppost to another, and when he had successively embraced and explored three of them, said "O, I shee, zis is a damned proshession"—everything explained, but hardly by that Real of yours. . . . The simplest notion of the Real . . . is History. And what is the prime character of History? Compulsion—symbolized by the traffic cop—No, this is still too intellectual—*being tackled at Rugby, there is the Real.* Nobody who hasn't been knocked down has the slightest notion of what the Real is . . . I used to play in the middle of the scrum. They used to hack at your shins to make you surrender the ball, a compulsory element—but the question was *How you took it*—your own self-creation. Freedom lies in summoning up a mentality which transforms the situation, as against letting organic reactions take their course.

In point of fact, I was in considerable agreement with this view of proximate Reality, thinking that the will to win in Rugby or elsewhere had more than a little explanatory power. And Whitehead was equally ready to welcome a convergence of views. When I brought out the "will to power" as central in the instinctive life

of man, he asserted "enthusiastic agreement: before this, I have been off and on with you." And this spirit of frank divergence as a way to better-founded convergence was typical of the experience of the seminary. The debate was real.

And the results were tangible. Each of us was ready to acknowledge certain changes of mind. At one of our March meetings Whitehead said to the Seminary,

> I am conscious of having started with a decided antagonism to the Royce-Bradley tradition, and of having in the course of time approached to it. . . . Its success lies in the wealth of intuitions expressed. But there is the great difficulty in verbalizing them, and making them consistent with one another. That is the task of this century.

As to the necessity of the system of categories, he remained dubious of the ideal, but said "I reserve the right to change my mind at any time, regarding that as a privilege of the seminary." As for me, I noted in the record "I have been led to a revision of the place of consciousness within mentality." And I doubt not that my growing sense of the inductive element in metaphysics, involving the judgment that the "a priori," constantly sought, is likely to be the last thing we arrive at, is in part a product of this experience.

The next year, we offered a second seminary on the same plan with the topic, "What do you mean when you appeal to experience?" Later we planned a third, which was set aside by a call of the Gifford Lectures in Scotland. These experiences with Whitehead were among the richest in my life as a teacher; I feel confident that they were helpful to students. They remain a treasury of informal dialogue with a mind of unique charm and resource.

Whitehead's way with students. In spite of the reputed difficulty of his discourse, his student audience continued to grow. His attitude was encouraging but never condescending. Collectively he credited his students with an advancement well beyond their actual attainment; no one got everything, but almost all responded to the spur. He sat there inviting us to follow his argument and helping us over hard spots with a rhetorical "You see . . . you see"—usually an assumption contrary to fact.

He sometimes had notes, but, as I recall, seldom stuck to them; he gave the impression of a mind not repeating former results but winning anew the insights he had to convey—it was water from a living spring, not from a faucet. It was at times as if he were speaking to himself, wrapped in the movement of the thought and

almost unaware of the group before him, they straining to hear the words which became less audible as his own wrestling became intense. One of his Radcliffe students tells me that on one occasion, as his voice drifted away toward soliloquy, he abruptly recovered himself with the words, "Excuse me, ladies; I forgot that you were there!"

In his criticism of their work, there was a generous interpretation of effort, but also an unsparing rigor in dealing with defects in his own line of fire. He was capable of a uniquely adroit union of friendly spirit with deadly analysis. One such occasion holds its own in my memory with almost verbal insistence:

It was toward the end of October, 1924. An able graduate student in Whitehead's first "seminary" had just returned from a year in Europe under a highly technical philosopher. He had read a lengthy paper, using much of that philosopher's terminology. I looked at Whitehead, who appeared studiously attentive, wondering how much he was absorbing. At the end, he addressed three accurately pointed questions to the reader, and then summed up his judgment:

> This careful paper reminds me of something. (An enquiring pause.) Oh yes! It is a passage in Tolstoy's *War and Peace*, dealing with the evening before the battle of Austerlitz. The joint Austrian and Russian staffs had met to prepare for the impending attack by Napoleon. The Austrian chief-of-staff was reading the dispositions for the next day; everything was provided for, to the last detail; it was a masterly affair. And Kutuzoff, the old Russian general, *went fast asleep.* On the next day Napoleon appeared, but in an entirely different quarter than had been anticipated in the admirable dispositions, which therefore *went for nothing!*
>
> Now, for our purposes, *Napoleon may represent reality!* When I hear a paper as intricate as this, I find myself very much in the position of the old Russian general. I say to myself, reality cannot be half so complicated! (This last, with the gentlest quietude.)

As I recall the event, this shaft, carried by the kindly humor of the analogy, went home with full force, just for that reason. The members sat for a moment in silence, partly in recognition of the aptitude of the comment, partly in sympathy with the reader (who received it with firm equanimity), and partly—in at least one hearer's mind—with an enquiring sense whether the implied criterion of a relatively uncomplicated reality is universal.

On this point, Whitehead had his own comments, as in an early lecture (Nov. 8, 1924), when he said:

> The dilemma of metaphysics is that either you are clear, and leave much out, or else you are adequate—and muddled . . . You come to a point where clearness is impossible . . .

For example (from a later lecture, Nov. 24):

> To meet the difficulty of secondary qualities . . . you can't say they are there in the object; you don't like to say they are in your head; and so you say they are in the mind! If so, you make Hume inevitable, and Kant. To avoid this you must recognize that the description of the given fact has been too simple . . .
>
> Mind is inside its images, not its images inside the mind . . . I am immersed in the topic, not the reverse. . . . We are actors in scenes—you and I in this scene—not the scene inside us. . . . This is not a horrid paradox, but a plain man on top of a bus . . . (Nov. 11).

No one has described the impact of Whitehead's teaching on the students more eloquently than one of those students, an editor of *The Crimson,* who wrote of him (Nov. 20, 1936):

> He teaches no dogma . . . [but achieves] an ignition between mind and mind . . . [such that] the torch lives on . . . as indestructible as philosophy itself.

The paradox of Whitehead's ultimate aim. It seems to me particularly important for our confused era to recognize that Whitehead himself was in some sense a union of opposites. No one could have used more scrupulous care to set up a system of categories, consistent and adequate to the full breadth of experience. No one has used more vigorous language in condemning not only the assumption but the hope of finality. If we ask whether any set of categories is a necessary set, we seem to have in him two answers: a firm yes, and an equally firm no.

His achievement is undoubtedly a *system,* a complex of categoreal principles, each of which is demanded for an adequate description of human experience in its most universal scope. Of the concept of God, primordial and consequent, he said to me:

> I should never have included it, if it had not been strictly required for descriptive completeness. You must set all your essentials into the foundation. It's no use putting up a set of terms, and then remarking, "Oh, by the by, I believe there's a God."

It is precisely the *necessity to which one feels him obedient* as he moves from point to point that constitutes the power of Whitehead's thought, lifting the entire level of contemporary speculation and felt constantly by all who were near him.

Yet it is Whitehead who repeatedly declares common conviction the final authority in metaphysics, and who in the final section of his last public lecture at Harvard—four years after his retirement—[7] twice uses the term "fake" to describe the claims of

[7] The Ingersoll Lecture, April 22, 1941.

"logic" and "exact statements" to serve as "adequate analysis of the advance of thought." "The exactness is a fake": these were his last public words to us.

One consequence of this striking coexistent contrast was, I think, the permanent companionableness of Whitehead, always meeting new questions with an unfeigned interest—a modesty in achievement which was entirely sincere. As a profound interpreter of the era of relativity, whether in physics or in morals, he was a living rebuke to the assumption that relativity implies an absence of ground to stand on and, therefore, a relaxation of all effort toward foundations. The impression, "There is no firm ground," comes near to being the dominating malaise of our time. Whitehead draws the opposite conclusion: the acceptance of nonfinality means not a letdown but a heightened task. The failure of formulated absolutes leaves the absolute requirement to think again toward that uncaptured reality which contains and guides the total meaning of existence.

With little concern for the philosophical mystics of past eras, he comes to a restatement of the aims of philosophy:

> If you like to phrase it so, philosophy is mystical. For mysticism is direct insight into depths as yet unspoken. But the purpose of philosophy is to rationalize mysticism.

His life and his felt dedication were a summons to the eternal discontent in which even mental despair confesses itself the mother of creativity—the unexplained first principle.

WILLIAM ERNEST HOCKING
HARVARD UNIVERSITY, EMERITUS

WHITEHEAD'S THEORY OF BECOMING

MY purpose in this paper is mainly critical: I want to show that one of the positions that Whitehead defends in his later philosophical writings is untenable. My criticism will be twofold. I shall claim, first, that the position in question itself is unintelligible and, second, that the argument on which Whitehead rests this position is invalid. My aim is not wholly critical, however, for I shall make some positive suggestions, in the light of my critical conclusions, regarding Whitehead's conception and practice of philosophy. Whitehead says that his method, in his later works, is that of "speculative philosophy," and he devotes some space to explaining what this is and to defending his conviction that it is

"a method productive of important knowledge." But I think it is not altogether clear, from either Whitehead's statements or his practice, just what speculative philosophy is. And since I think that a good deal of the contemporary disregard, if not outright disdain, of Whitehead's later philosophical work is founded on an ignorance or misunderstanding of the business that Whitehead has set himself to do, I think it is worthwhile to try to make this plain. I think more contemporary philosophers might read Whitehead and appreciate his genius if they knew better how he conceived the philosophical enterprise. Of course there are some who will say that the task of speculative philosophy, as Whitehead understood it, is not one that ought to be or needs to be or even can be done and, hence, that Whitehead's later work is beside any properly philosophical point. I think it would be wrong to say this and that philosophers who do or would say it have too narrow a conception of philosophy themselves, that they arbitrarily limit the scope and power of philosophy. But I shall not here undertake to defend this point; I shall not try to defend Whitehead's claim that speculative philosophy is a method productive of important knowledge. I shall confine myself rather to the task of making clear, at least in part, what speculative philosophy is for Whitehead.

The Whiteheadian position that I want to criticize is the "epochal theory of time," or, as Whitehead might better have called it, of becoming (SMW 183–186; PR 105). According to this theory, time (or becoming) "is not another continuous process [but] is an atomic succession" (SMW 185). This means that particular processes, particular stretches of time or becoming, cannot always be divided into divisible processes, for at some point the division will reach the atomic units of these processes, the units whose succession constitutes these processes. But now Whitehead holds that these atomic units of process are themselves processes, though of a quite different kind. The composite sort of process Whitehead calls "macroscopic," or the "process of transition"; the unit sort he calls "microscopic," or the "process of concrescence" (PR 320 ff.). Hence the epochal theory can be more accurately stated as follows: A process of transition is a succession of processes of concrescence; the former process is discontinuous, because division of it will eventually yield its component processes of concrescence, and these component processes are themselves indivisible. The process of transition is divisible, but not *in infinitum;* it is not endlessly divisible into sections that are themselves divisible processes of transition. But the process of concrescence is not

divisible at all and, hence, is neither continuous nor discontinuous; this contrast has no application to this sort of process.[1]

According to Whitehead, the process of concrescence is the process by which an actual occasion comes into being; it is the act of becoming that constitutes the actual occasion, that is its "real internal constitution." Hence a process of transition is a succession of actual occasions. Now it is clear that the discontinuous character of the succession is owing to the atomic or indivisible character of the occasions, not the other way around. Hence the fundamental fact to which the epochal theory alludes is the indivisibility of actual occasions, i.e., the indivisibility of the processes or acts of becoming of which actual occasions are constituted. Whitehead sometimes expresses this point by saying that the act of becoming that constitutes an actual occasion "is not extensive" (PR 107), i.e., is not temporally extended. He even says that the act "is not in physical time" (PR 434).

But now, although the becoming of an actual occasion is indivisible and not extensive, according to Whitehead, the occasion that becomes, the completed entity, is temporarily extended and so is divisible. The same holds for the completed succession of actual entities that is the creature of a process of transition; it too is divisible, and divisible *in infinitum*. Thus there is a contrast for Whitehead between the *product* and the *process* of becoming, whether microscopic or macroscopic. The former is extensive and continuous for both sorts of becoming; the latter is discontinuous (in the case of macroscopic becoming) or not extensive (in the case of microscopic becoming). Or as Whitehead also puts it, "there is a becoming of continuity, but no continuity of becoming" (PR 53). The same contrast is expressed in Whitehead's double use of the word "time." Sometimes he uses it to refer to the *product* of a process of (macroscopic) becoming (so, e.g., in SMW 183), sometimes to the *process* itself (so, e.g., in PR 95, 434).[2]

The conclusions that constitute the epochal theory of time, Whitehead says, "are required by the consideration of [certain philosophical] arguments" (PR 105; cf. SMW 186), in particular of one of the celebrated motion paradoxes of Zeno. Zeno's arguments are not all valid in Whitehead's view, but there is one of

[1] The process of concrescence can, however, be *analyzed* into its component *prehensions;* see, e.g., PR 35 and *passim.*

[2] Whitehead uses the word in both ways and explicitly contrasts the two uses in his article "Time" (*Proceedings of the Sixth International Congress of Philosophy,* 1926), pp. 63–64.

them which is; Whitehead refers to this as the "Arrow in Its Flight," but it is actually the Dichotomy that he has in mind, as his discussion of it makes clear. As Aristotle states it, Zeno's argument is "that in traversing any distance we must first traverse the half of it, that these subdivisions are infinite, and that it is impossible to pass through an infinite number of distances [*sc.* in a finite time]."[3] But Aristotle's statement is ambiguous; he fails to make clear whether the "infinite number of distances" to be traversed by any moving object were meant by Zeno to form the series ". . . , ⅛, ¼, ½," i.e., a *regression,* or the series "½, ¼, ⅛, . . . ," i.e., a *progression.* Aristotle himself supposed the latter and hence was able to say that Zeno's Dichotomy argument "is essentially the same as" the so-called Achilles (*Physics* Z, 9, 239b18–19). But Simplicius and numerous commentators after him chose the former alternative.[4] For them the difficulty that Zeno's argument uncovers is not that the motion of a moving body cannot be *completed,* but rather that it cannot *start,* that no body can *begin* to move. For—so the argument runs—a body cannot move without traversing some distance; but before any distance can be traversed there is always another distance, *viz.,* the half of the original distance, which must be traversed first; and since there is therefore no distance that can be said to be *the* first, there is also no movement that can be said to be *the* first, which means that motion cannot begin. On this interpretation the Dichotomy is not the same as the Achilles.

It is clear that Whitehead interprets Zeno's argument as Simplicius did and not as Aristotle did; hence he stresses the difference between it and the Achilles (PR 107). But Whitehead also changes the argument, so interpreted, in at least two ways. First,

[3] *Physics* Θ, 8, 263a4–6; see *ibid.* Z, 2, 233a23 for the scholium. I have used the translation of Aristotle that is given by H. D. P. Lee in his book *Zeno of Elea* (Cambridge: University Press, 1936), except that I have used "pass through" for "διεξελθεῖν" and "διεξιέναι" in place of Lee's "complete," which seems to me to make for awkward English. I have, however, preserved Lee's distinction between these two terms on the one hand and "διελθεῖν" and "διιέναι" on the other (a distinction not observed by the Oxford translators) by retaining Lee's "traverse" in rendering the latter two terms.

[4] See Simplicius's commentary on the *Physics,* 1013.4 ff. More recent commentators who take the same view are, e.g., V. Brochard, "Les arguments de Zénon d'Élée contre le mouvement," *Séances et travaux de l'académie des sciences morales et politiques,* n.s. **29** (1888), 555–568; G. Noel, "Le mouvement et les arguments de Zénon d'Élée," *Revue de métaphysique et de morale,* **1** (1893), 107–125; J. Burnet, *Early Greek Philosophy* (London: Black, 1892; 4th ed., 1930); Lee, *op. cit.;* and W. D. Ross, *Aristotle's Physics* (Oxford: University Press, 1936).

he generalizes it; he makes the argument apply not just to motion but to becoming generally, which is to say to all forms of change and process and even to mere persistence through time, since on Whitehead's view persistence is simply a succession of distinct becomings.[5] Second, Whitehead adds a premise to Zeno's argument. As it stands, the argument assumes that any process of motion (or, in its generalized form, of becoming) is divisible into parts that are themselves divisible processes of motion (or becoming). But Zeno's conclusion does not follow, Whitehead says, unless it is also assumed that "in a becoming something (*res vera*) becomes" (PR 106)—and indeed, as a later passage makes clear, a "something with temporal extension" (PR 107).[6] So generalized and supplemented, Zeno's argument is valid, according to Whitehead; it "elicits a contradiction" from the two assumptions mentioned. That is, it follows from the argument *either* that in a process of becoming nothing (having temporal extension) becomes *or* that the becoming is not always divisible into parts that are themselves (divisible) becomings (PR 106). And this means that one of these two assumptions must be given up. Whitehead might have given up the first, as Bergson did; he might have regarded the cosmos as something that, in Plato's phrase, "is always in the process of becoming . . . and never really is." But this goes against Whitehead's fundamental rationalism; for a world without real individuals, each with a distinct identity and a definite character, is not a world that can be rationally understood—which of course is just Bergson's point. In any case, Whitehead did respond to Zeno's argument by giving up the second of the two assumptions noted above. The argument is valid, Whitehead holds, and something (with temporal extension) does become in a process of becoming; hence we are forced, in his view, to deny that every process of becoming can be divided into parts that are themselves (divisible)

[5] Hence his statement that "the true difficulty" brought to light by Zeno's argument "is to understand how [anything] survives the lapse of time" (PR 106).

[6] The qualification is essential, Whitehead says, for Zeno's "difficulty is not evaded by assuming that something becomes at each non-extensive instant of time" (PR 106). But Whitehead does not always state the qualification in stating the premise. Indeed in SMW he states Zeno's argument, and says that it is valid, without this added premise; see page 186. But this must be a slip, for Zeno's argument is obviously invalid without this added premise; see below, page 523 and footnote 8. It should be noted that the premise Whitehead adds to Zeno's argument can be added only after the argument has been generalized and made to apply to becoming. For there is no comparable premise, no analogous addition, that can be stated for the case of motion by itself.

becomings. Zeno's argument *shows* that some becomings are indivisible, according to Whitehead; better, it shows that macroscopic becomings are discontinuous and, more fundamentally, that microscopic becomings are not extensive. And this, as we have seen, is just the epochal theory.

I turn now to my criticism of this theory. I shall first argue (1) that the theory itself is unintelligible, and then try to show (2) that the Zenonian argument on which the theory is founded, even in its amended, Whiteheadian version, is invalid.

(1) The epochal theory stands or falls with the notion of a becoming that is not extensive. For the act of becoming that constitutes an actual occasion is indivisible because it is not extensive, and the process of transition whereby each actual occasion gives way to its successor is discontinuous because its component concrescences are indivisible. But the notion of a becoming that is not extensive is an odd notion at best. Becoming is ordinarily thought of as a process that takes a certain time to occur and that extends, therefore, through a certain period of time. To say that a certain act or process of becoming is not extensive, however, would seem to imply that the becoming does not take any time to occur, that it is an act or process that extends through no time. The becoming may still occur at a time, i.e., at a certain instant, but only at that instant; hence it will be an instantaneous becoming. And it may be wondered how any act or process could be instantaneous, how it could (logically) take no time to occur. Whitehead indeed seems to be aware of this difficulty, for he seems to be pulled in two opposite directions in speaking about the process of concrescence. On the one hand he says things about it that seem clearly to imply its temporal extendedness. It can be "analyzed genetically into a series of subordinate phases which presuppose their antecedents" (PR 234); these phases are "successive" (PR 249); they are "real genetic phases" which together effect "the growth from the real to the actual," the growth "proceeding from phase to phase, each phase being the real basis from which its successor proceeds toward the completion of the [actual entity] in question" (PR 327). On the other hand, Whitehead says things that imply that the process of concrescence is not temporally extended, things over and above the original statement that it is not extensive. He says, e.g., that this process "is not in physical time" and that it "is not the temporal succession" (PR 434).

But even if this difficulty could be met and a sense found or provided for the notion of an instantaneous becoming or of an act or process that takes no time, there is another difficulty to be

faced. Concrescences, or acts of microscopic becoming, *take* no time in Whitehead's view; they could not do so and still be non-extensive. But they must occur *in* time; they must have dates even if they are without duration. But the events or occasions that are the products of such acts are also located in time. Indeed, they have both date and duration, according to Whitehead; the existence of each begins at one definite time, ceases at a later time, and extends through all the times between. But if both the act and the product of the act of becoming are in time, there must be some temporal relation between them. But what could this relation be? Certainly the act cannot be either altogether earlier or altogether later than the product; the act's date must coincide with one of the times through which the produced event exists. But we cannot say that the act is simultaneous with the event's time of cessation, for then we should have to grant that the event began before its act of becoming had occurred, which is absurd. Nor can we, for the same reason, say that the act occurs at any time later than that at which the event begins to be. Therefore we must conclude that an actual occasion's act of becoming coincides with the moment at which the occasion begins to exist.

But this conclusion will not stand. An event or occasion is said to exist from time t_0 to time t_1 because an act of becoming, the act whereby it comes into being, occurs at t_0. But what does it mean to say that an event or occasion comes into being? *Things* come to be, but events *occur* or *happen*.[7] Yet the products of acts of microscopic becoming *are* events or occasions according to Whitehead, and are not things or objects. What Whitehead must mean, therefore, when he says that actual occasions become, is just that they happen; or at any rate he must allow that what "becoming" signifies, when used of actual occasions, is no different from what we should ordinarily call "happening." Substituting "happening" for "becoming," then, the above conclusion reads: an actual occasion exists from t_0 to t_1 because it happens at t_0; the occasion, which is temporarily extended, begins at the time of its happening, and its happening is itself instantaneous. Put in this way, however, Whitehead's doctrine is easily shown to be senseless. For it requires us to distinguish the happening of an event or occasion from the event itself, or from the event's existence. But how does an event differ from its happening, or how does its existence do so? The answer is that it differs not at all. An event is not one thing and its happening another; happening adds nothing to an event, as being or existing adds nothing to a thing or object. An

[7] Cf. J. J. C. Smart, "The River of Time," *Mind*, **58** (1949).

event, in short, *is* its happening; to be an event and to happen are one and the same.

It follows that there is a crucial difference between happening and becoming in the proper sense and also between objects or things and events. For the being or existence of a thing is distinct from its becoming, but the "being" of an event just is its occurrence or happening, so that in the case of events "being" and "becoming" coincide. Things (objects) come into being and then are; their becoming precedes their being; and although nothing can come into being without thereby being, it is at least conceivable that a thing should be and not have come to be. But if an event "becomes" by happening it also "is" by happening. Then if we choose, with Whitehead, to use "becoming" of events or actual occasions, we must acknowledge, as Whitehead himself does not, that our meaning is no different from what it would have been had we used "being" instead.

Actual occasions, Whitehead says, are temporally extended, but their acts of becoming, which is to say their happenings, are not. But if an occasion is its happening, if it exists by happening, then Whitehead's doctrine ends in a contradiction: one self-same thing both is and is not extensive. And this is sufficient to show the unintelligibility of Whitehead's view.

(2) But not only is Whitehead's epochal theory of becoming unintelligible; the argument on which it is founded is invalid as well. It is easy enough to show the invalidity of Zeno's Dichotomy argument in its original form, i.e., as it stands without the changes that Whitehead made in it. For in this form the argument assumes that because there is no *first* distance traversed by a moving body and hence no *first* motion, its motion can have no *beginning*. And this is clearly not the case. As Adolf Grünbaum has expressed it,

> . . . what is true in the "Dichotomy" argument as applied to the theory of events is that the dense ordering of the events [required by the "infinite divisibility of time"] entails the absence of a first subinterval among the continuum of events which elapse. . . . [But] the requirement that there be such a first subinterval [is] entirely gratuitous. There is a first event in the motion process but no first motion, no first time interval and no first space interval. . . . Zeno was entirely successful in showing that there is no first motion but it does not follow that there is no motion, as he thought. . . . there is no more reason why there should be [a first motion] than there is a reason for the existence of a first rational fraction greater than 0.[8]

[8] "Relativity and the Atomicity of Becoming," *Review of Metaphysics*, **4** (1950–1951), 177–178. Cf. Aristotle, *Physics* Z, 5, 236a14 and 26–27; also *ibid.*, Z, 6, 237b5–7; and R. M. Blake, "The Paradox of Temporal Process," this JOURNAL, **23** (1926), 653.

So the question before us is this: are the changes Whitehead makes in Zeno's argument sufficient or of the right sort to change it from an invalid argument to a valid one? And the answer is that they are not.

Whitehead changes Zeno's argument in two ways: (1) he makes it apply not just to motion but to becoming generally; (2) he adds the premise that "in every act of becoming there is the becoming of something with temporal extension" (PR 107). He then draws the conclusion that macroscopic becoming is discontinuous because composed of acts of microscopic becoming that are indivisible, on the grounds that if there were not such indivisible acts nothing having temporal extension would become in an act of becoming, which controverts the added premise. Whitehead must mean, therefore, that in every act of *microscopic* becoming, in every process of concrescence, there is the becoming of something with temporal extension. To use the terminology of a recent writer, Whitehead's view seems to be that "becoming," in the microscopic sense, is a "verb of accomplishment," i.e., that actions which count as becomings have a "set terminal point, . . . a 'climax,' which has to be reached if the action is to be what it is claimed to be"; such actions "proceed toward a terminus which is logically necessary to their being what they are."[9] Thus there is no becoming until the terminus has been reached and the accomplishment—*viz.*, the creation of something with temporal extension—has been effected. Becoming cannot then be divided (into becomings), for the reason that the products of any division are logically precluded from being themselves becomings; to say that part of a becoming is itself a becoming would be to go against the meaning of the term "becoming."

But let us consider the nature of the accomplishment in the case of (microscopic) becoming. Becoming, Whitehead says, must issue in something with temporal extension. He does *not* say, something with a certain *minimum amount* of temporal extension. Nor does he mention any other condition that a thing must meet to count as an accomplishment or product of an act of becoming. Of course, actual occasions, the products of acts of microscopic becoming in Whitehead's view, do have other features, and features essential to their being what they are. But these features are not mentioned in Whitehead's statement of the premises of Zeno's argument. The conclusion is to follow from the sole requirement that what becomes have temporal extension; the only condition necessary for some-

[9] Zeno Vendler, "Verbs and Times," *Philosophical Review*, **66** (1957), 145–146.

thing's being a product of an act of becoming that is relevant to the conclusion is this, that the thing in question be temporally extended. But if a thing need only be temporally extended in order to be an accomplishment of an act of becoming, there is no reason to deny that part of such a thing is itself such an accomplishment. Half a temporally extended thing is still a temporally extended thing, though it is not of course the original whole thing. Let us now suppose that an act of becoming, which by its nature must issue in some temporally extended thing, were itself temporally extended and so divisible into parts. There is then no reason to deny the existence, for each resulting part of the act, of a corresponding part of the accomplishment of the whole act, which, since it is a temporally extended thing, will also be divisible into parts. Nor is there any reason to deny that each such part of the accomplishment is *the* accomplishment of the corresponding part of the act. And finally, since each part of the accomplishment is itself something with temporal extension, there is no reason to deny that the part of the act of becoming whose accomplishment it is is itself an act of becoming. For an act of becoming need only, on the grounds actually stated by Whitehead, issue in something with temporal extension in order to count as an act of becoming.

I conclude, therefore, that the conclusion drawn by Whitehead from Zeno's Dichotomy argument, even as Whitehead amends it, does not follow; it does not follow from the premise that in an act of microscopic becoming something with temporal extension becomes that such an act is itself indivisible, not extensive, and "not in physical time." Zeno's argument is no more valid with the changes made in it by Whitehead than it is without them.

I want now to make some general points about Whitehead's philosophical procedure which are suggested and I think illustrated —though they are not of course established—by the results of my critical examination of the epochal theory. To begin with, I want to note how little of Whitehead's philosophical position as a whole is affected by my criticisms. If my claims are just, the epochal theory of time or becoming is both untenable and unnecessary; there is no sense to the doctrine that the process of concrescence or act of becoming that constitutes an actual occasion is temporally nonextensive, and no reason to hold such a doctrine. But this result, far from striking at the heart of Whitehead's philosophical position, only cuts off a minor appendage, an appendage which has no function other than that of protecting Whitehead's system from

the—supposed—upshot of Zeno's argument, and which has no ground in or connection with any other element in the system. There are four major and essential tenets in Whitehead's position regarding actual occasions: (1) that there are real individuals with distinct identities and definite natures and that actual occasions are such individuals; (2) that the fundamental units of reality are temporally extended and, hence, that actual occasions are; (3) that actual occasions exist by becoming, that they are self-creative processes; and (4) that actual occasions are finally caused and hence have an atomic character, which is to say that the process that constitutes an actual occasion cannot be divided into processes that are themselves actual occasions. No one of these tenets is in the least affected by my criticisms of the epochal theory. Each of them is logically independent of the epochal theory and is held on grounds other than Zeno's argument—each indeed is held on grounds of a different sort from those provided by Zeno's argument, as I shall suggest in a moment. It might be thought that (4) above *is* the epochal theory, but this is not the case. For the epochal theory is not that the process of concrescence is indivisible in *any* way for *any* reason, but that it is indivisible in the *particular* way and for the *particular* reason that it is temporally nonextensive. And my criticism of the epochal theory was directed not against the notion that there are indivisible processes or that actual occasions are indivisible processes in general—because finally caused or for any other reason. My criticism was directed rather at the notion of a *temporally* indivisible process, the notion that actual occasions are indivisible because temporally nonextensive. I concluded that there is no reason to deny that actual occasions are temporally extensive and hence temporally divisible—divisible into parts that are themselves becomings of something with temporal extension. But this leaves altogether open the possibility that actual occasions may be indivisible in some other way and for some other reason. And Whitehead's basic reason for holding that actual occasions are indivisible unities, atomic drops of process, is, I think, his doctrine that they are finally caused, which doctrine is in no way required by or even related to the Dichotomy argument of Zeno.

The fact that the major elements in Whitehead's account of actual occasions are independent of Whitehead's own use of Zeno's argument and, hence, are untouched by my criticism of that argument and its alleged conclusion suggests that the role played by such arguments in Whitehead's philosophy is relatively slight. Since Zeno's argument is typical of many philosophical arguments

and indeed is a paradigm of a certain important type of philosophical argument, we may say that the role played by philosophical arguments of this type in Whitehead's philosophy is small. To be sure, Whitehead often cites such arguments; he often mentions, develops, states, and even employs them in his writings. But their function is not, I believe, to *determine* the elements of his philosophical system, to settle *which* positions shall find a place in it—though this is the function that they generally appear to have and that Whitehead himself perhaps thought they did have. Their function is rather to provide a kind of negative *test* of positions already arrived at by quite other means, and in particular by non-argumentative means. Whitehead's philosophical position is not so much the *conclusion* of a philosophical *argument* as an *imaginative construction*. This construction must, Whitehead says, meet certain tests, some internal or "rational," and some external or "empirical" (PR 4 ff.). Among the external tests Whitehead includes only (1) that the construction be "applicable," and (2) that it be "adequate." But another external test that Whitehead actually employs, and employs frequently, is that the construction be secure against traditional philosophical objections, the objections that arguments such as that of Zeno bring to light. Hence philosophical arguments do have a role in Whitehead's philosophy, but a subsidiary and secondary role. They are used by Whitehead, at least for the most part, not to determine what goes into the philosophical system, but to help make what has already been put there comfortable and safe. Hence they play their role only after the main content of the system has been settled upon and settled upon by altogether different means.

The fact that philosophical arguments, at least of the type represented by Zeno's argument, play this subsidiary and relatively minor role in Whitehead's philosophy is one thing, I think, that makes Whitehead's later work uninteresting at best to many contemporary philosophers. For the majority of present-day philosophers, or anyhow for those of an "analytic" bent, a philosophical doctrine is essentially the conclusion of a philosophical argument; philosophy is principally and distinctively argumentative or dialectical in character, and imaginative construction, whatever its own merits, is not a *philosophical* procedure—its use does not result in *philosophical* theses. And it is pretty obvious, I think, that the major tenets of Whitehead's philosophical position are not the conclusions of philosophical arguments, for all that Whitehead does make use of such arguments. The fact that his use of them is often careless and unsuccessful, as I have argued it is in the case of Zeno's

argument, has seemed to many philosophers to confirm their low judgment of Whitehead's philosophical prowess. But I think that in fact it does nothing of the kind. For not only do I think that Whitehead's use of philosophical arguments is different from the use these philosophers would make of them and that such arguments are of less importance to him than to them. I also think that their conception of philosophy, as argument and little else, is unwarrantedly narrow. Philosophy also, to my mind, comprehends the sort of imaginative construction that was Whitehead's forte and that he practiced better, as I believe, than anyone else has done in the present century. But this is itself a claim that requires justification, and I have not tried to provide such justification here. I have not even tried to justify my claim that philosophy for Whitehead *is* essentially imaginative construction, although this too needs justifying. For there are some students of Whitehead's philosophy, I imagine, who might dispute it.

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STRAINS

ACCORDING to Whitehead, actual entities function in the becoming of other entities (PR 38). An entity functions in the becoming of another if it contributes position or definiteness or both to the other entity. Each actual entity, feeling, nexus, and generally any entity that becomes has associated with it an extensive quantum. Also, each of these entities may be said to occupy the volume of its quantum. The volume of an extensive quantum is the set of all the interior points of the quantum. The position of entities is specified by the relative position of the points of their volumes. But a point is a special set of sets of entities, and the relative position of points is specified by the topology of the space of all points. In its turn, the topology of this space is specified uniquely by the volumes of the extensive quanta. The entities that become occupy their volumes, volumes are located relative to one another by their points, and the relative position of points is determined by the volumes of these entities. Thus we come full circle and may say that the entities that become position one another.

In a similar way, we may say that entities contribute to one another's definiteness. The definiteness of an entity is, for Whitehead, the exemplification by that entity of selected eternal objects. It would seem that, besides the entities and the sets of entities that become, our universe must also contain an entirely different kind of entity that is eternal. But it will be seen that we may construe

eternal objects as special sets of becoming entities. These sets have the property of being independent of position. An entity that becomes will be said to exemplify an eternal object when it is a member of the object. If we conceive eternal objects to be special kinds of sets of entities, then we may say that the entities that become mutually determine one another's definiteness as well as one another's position.

Even though entities determine one another's definiteness and position, these two features of entities are relatively independent of each other. Becoming entities acquire position through the agency of physical feelings, and they acquire definiteness through the agency of both physical and conceptual feelings. But neither does the position of an entity uniquely determine its definiteness, nor does the definiteness of an entity uniquely determine its position. Consider, for example, the case of a physical feeling. Among the eternal objects exemplified by feelings are the geometric elements. A given geometric element, say a specific flat locus of points contained in the volume of some feeling, is a special set of sets of sets of entities that is not eternal. On the other hand, flatness is an eternal object among whose members is the flat locus in question. Thus, many other feelings could exemplify flatness, but each such feeling would have a different position.

Yet position and definiteness are not absolutely independent features of every entity. For example, physical feelings must reproduce the definiteness of their objective data; hence every eternal property of the geometric elements of the data is reproduced in the feeling. If flatness is exemplified in the data, then the corresponding feelings may not occupy volumes that do not contain flat loci of points. Conversely, if the feelings occupy volumes that contain flat loci, then they must exemplify flatness.

There follows a brief sketch of the theory of feelings, together with an account of the connection between this theory and the theory of extensive abstraction. The purpose of this sketch is to obtain definitions of "definiteness" and "position." In terms of these two concepts we shall elucidate the reproduction of eternal objects in physical feelings. The reproductive character of physical feelings will be illustrated by the particular case of strains. Since a strain is a physical feeling whose objective datum is dominated by flat loci of points, it will serve well to illustrate the reproduction of geometric eternal objects (PR 472). Furthermore, the other eternal objects exemplified by the datum of a strain may be assigned to flat loci so that the corresponding flat loci of the strain will be qualified by the same eternal objects as the flat loci of the datum (PR 441).[1]

[1] Such an assignment of eternal objects to flat loci is what Whitehead calls a "projection."

Let S be the set of all feelings, S_1 be the set of all physical feelings, and S_2 be the set of all conceptual feelings. $S = S_1 \cup S_2$ and $S_1 \cap S_2 = \Lambda$. "F" is to be read "is a physical feeling of." F is a many-one, asymmetric, and irreflexive relation whose domain is S_1 and whose range is S. "R" is to be read "has the same subject as." R is an equivalence relation whose field is S. A is the set of all equivalence subsets of S with respect to R: $A = S/R$. The members of A may be called "actual entities." Some of the conditions satisfied by F, R, and A are as follows:

1. If $X \in A$, then $X \cap S_1 \neq \Lambda$ and $X \cap S_2 \neq \Lambda$.
2. If Rxy, then neither Fxy nor Fyx.
3. If Rxy and $x \neq y$, then there is no z such that Fxz and Fyz.
4. If $X, Y \in A$ and if there is an $x \in X$ and a $y \in Y$ such that Fxy, then there are no $x' \in X$ and $y' \in Y$, $x' \neq x$, $y' \neq y$, such that either $Fy'x'$ or $Fx'y'$.

The categories of subjective unity and subjective harmony seem to require that R be an equivalence relation. If different subjects had common feelings, then we could surely abstract these feelings from their subjects. The category of objective diversity seems to require that no feeling have two distinct objects. Thus, F must be a many-one relation. The category of objective identity seems to require that no two distinct feelings of one actual entity have the same object (PR 39–41). Hence the relation F is one-one from the physical feelings that are members of one actual entity to the members of some subsets of S.

The notion of "actuality" explicated so far is nonrelative. A relative notion of actuality may be explained in terms of "Pre." "Pre," to be read "prehends," is defined as follows: If $X \in A$ and $Y \in A$, then Pre XY if and only if there is an $x \in X$ and a $y \in Y$ such that Fxy. Since the set A and the relation Pre form a lattice without upper or lower bounds (PR 33, 95–126, 296, 486–489), we may introduce a relative concept of actuality as follows: Y is actual for X if and only if Pre XY. The set of all Y that are actual relative to X is the actual world of X (PR 34, 117–126, 294, 345–346, 351, 435–439). The actual world of X is a subset of A that has no terminal member relative to Pre. According to the fifth category of explanation, the actual worlds of distinct actual entities are distinct (PR 33–34). The actual world of Y is a subset of the actual world of X if and only if Pre XY. The actual worlds of any two actual entities have a non-null intersection.

In order to explicate "position" we shall make use of some of the predicates of the theory of extensive abstraction. Among the predicates that we shall use are the following: "is a part of," "over-

laps," 'is a point," "is a flat locus," "is the volume of," and "is an ovoid" (PR 449–471). These predicates are defined by Whitehead in terms of the sole primitive predicate "is connected with." To these predicates we shall add two more: (*a*) "x is the aggregate of X" is to mean "any y overlaps x if and only if y overlaps some member of X"; and, (*b*) "x is a connected entity" is to mean "there exist no y and z such that y is not connected with z, and x is the aggregate of the set whose only two members are y and z."

The field of the relation *is connected with* is constituted by the members of S and the aggregates of any subsets of S (PR 441–444). If $X \epsilon A$ and x is the aggregate of X, then we shall say that x is the *satisfaction* of X. We assume that satisfactions are connected entities and that every connected part of a satisfaction is a member of S. Every satisfaction is a connected entity, and so is every feeling.

Let S_3 be the set of all satisfactions; then we have that the aggregate of S_3 is identical with the aggregate of S. Since R is an equivalence relation, no two distinct members of S_3 may overlap. In contradistinction, every member of S is overlapped by an infinite number of members of S. Indeed, every member of S must be a member of some abstractive set. In short, we suppose that the axioms of the theory of extensive abstraction are sufficient to show that every member of the field of the relation *is connected with* has a unique volume.

If every member of S and every aggregate of subsets of S has a unique volume, then all these volumes will be subsets of the volume of the aggregate of S. Let v be the volume of the aggregate of S, and let V be the set of all the volumes of the aggregates of subsets of S; then it may be ascertained that the ordered pair (v,V) is a normal topological space. This space is metrizable just in case there exists a denumerable subset S' of S such that every member of S is the aggregate of some subset of S'.

Once a metric for the space (v,V) is fixed, the distance between two points will receive a unique physical interpretation in terms of the relations that hold among the feelings whose volumes contain the given points. Any metric for the space (v,V) must have the property that the sets of points that are open in the sense of the metric are all and only those sets which are members of V. Since the members of V are the volumes of feelings and aggregates of feelings, the relative position of points is dependent only on the extensive relations that hold among feelings. The relative position of points is independent of the choice of a metric. Further development of the theory of position depends on the choice of the metric for the space (v,V).

Feelings may be said to occupy or to be positioned at their volumes (PR 458). Actual entities may be said to occupy or to be positioned at the volumes of their satisfactions. Similarly, we may say that sets of feelings or sets of actual entities occupy or are positioned at the volumes of the corresponding aggregates. Since the entities that are actual relative to some $X \epsilon A$ are all of those which are members of the actual world of X, we may say that the position of some actual entities is determinate relative to X if these entities are members of the actual world of X. If $A_j \subseteq A$,[2] then the members of A_j are *positioned relative* to X if and only if A_j is a subset of the actual world of X. If x is a feeling and $S_j \subseteq S$, then the members of S_j are positioned relative to x if and only if there is an $X \epsilon A$ such that $x \epsilon X$ and every actual entity whose intersection with S_j is non-null is in the actual world of X. Similar definitions of "relative position" may be given for aggregates, set of aggregates, and sets of sets of aggregates of subsets of S.

For the sake of simplicity we shall consider only those eternal objects which are subsets of the field of the relation *is connected with*. An eternal object is a subset of the field of *is connected with* such that its members are not positioned relative to any feeling. If S_j is a subset of the field of *is connected with*, then S_j is an eternal object if and only if there is no $x \epsilon S$ such that the members of S_j are positioned relative to x (PR 38, 69–70, 283–287, 364–365, 443–447).

In our basic language there is a rule of subset formation which states that, given any sentential function "F" in which "S_j" and "S" do not occur freely and given any set S, there exists a set S_j whose members are just those members of S which have the property F. F is independent of position if the set S_j is an eternal object. F is localized, or dependent on position, if S_j is not an eternal object.

The definiteness of an entity x may be conceived as the collection of all the properties of x that are independent of position. If $x \epsilon S$, then D_x is the form of definiteness of x if and only if the members of D_x are all and only those eternal objects S_j such that $x \epsilon S_j$. Since we are considering only feelings, the form of definiteness of x is a set of subsets of the field of the relation *is connected with*.

A physical feeling may be considered to be a transformation that reproduces the form of definiteness of the objective datum. This transformation is complex, since we require that every connected part of a feeling be again a feeling. If v_x is the volume of x and v_y is the volume of y, then (v_x, V_x) and (v_y, V_y) are the topological spaces induced by the identity transformation of v_x and v_y into v. If the set v is the volume of the aggregate of S and (v, V) is the space

[2] To be read: "A_j is either a proper subset of or equal to A."

previously defined, then V_x is a set of subsets of v_x such that every member of V_x is the intersection of v_x with some member of V. Similar remarks hold for (v_y,V_y). If Fxy, then there must exist a continuous transformation of (v_y,V_y) onto (v_z,V_z) such that the non-localized properties of y and parts of y are transmitted onto x and corresponding parts of x. More precisely, we assume that:

5. If Fxy, then there exists a homeomorphism H of (v_y,V_y) onto (v_x,V_x) such that, for any z, v, and D, z is a part of y, v is the volume of z, and D is the form of definiteness of z, if and only if there is a w part of x, $H(v)$ is the volume of w, and D is the form of definiteness of w. (Note: the relation *is a part of* is here taken to be reflexive.)

The feeling x conforms to its objective datum y in the sense that the forms of definiteness of x and y are identical and the forms of definiteness of parts of x and parts of y that have corresponding volumes are likewise identical (PR 361–367, 441, 445–447). If p_x and p_y are points of v_x and v_y such that $H(p_y) = p_x$, then p_x and p_y will belong to the volumes of parts of x and y that have identical forms of definiteness. Similarly, if $v'_x \subseteq v_x$, $v'_y \subseteq v_y$ and $H(v'_y) = v'_x$ (i.e., v'_x is the image of v'_y under H), then v'_x and v'_y are subsets of the volumes of parts of x and y that have identical forms of definiteness. Thus, to every point $p_y \in v_y$ and to the corresponding points $p_x = H(p_y) \in v_x$ we may assign the set of all the forms of definiteness of parts of y such that p_y is a member of their volume. More generally, to every set of points $v'_x = H(v'_y) \subseteq v'_x$ we may assign the set of all the forms of definiteness of parts of y such that v'_y is a subset of their volume. If we let Dv'_y be the set of forms of definiteness that is assigned to v'_y, then we may say that Dv'_y is the set of qualities implicated in v'_y (PR 472–473). $Dv'_y = Dv'_x$, if $H(v'_y) = v'_x$.

Every sufficiently small part of y whose volume contains v'_y is an *ovoid* if an only if v'_y is a flat locus of points. If the set of all ovoids is an eternal object, then v'_y is a flat locus if and only if the set of all ovoids is a member of every member of a certain subset of Dv'_y. Thus, if $v'_x = H(v'_y)$ and v'_y is a flat locus, then so is v'_x.

If there is a collection G_y of flat loci of points that are subsets of v_y and such that any subset of v_y is the union or intersection of some members of this collection, then we may say that v_y is *dominated* by G_y (PR 472–473). If v_y is dominated by G_y and Fxy, then v_x is dominated by a corresponding collection G_x of flat loci, where $v'_y \in G_y$, if and only if $H(v'_y) = v'_x \in G_x$.

A feeling x whose objective datum y has a volume v_y dominated by a collection G_y of flat loci is called by Whitehead "a strain." The form of definiteness of the feeling x or of a part of x may be expressed in terms of the set of qualities implicated in the volume

of x or in the volume of parts of x. Every volume of a part of x, where x is a strain, must be the union or intersection of a collection of flat loci that are members of G_x. The members of G_x and G_y are paired in one-to-one fashion. Consequently, the form of definiteness of the strain x may be expressed in terms of the set of qualities implicated in G_y.

Since corresponding parts of a physical feeling and its objective datum have identical forms of definiteness, the volumes of these corresponding parts must have identical nonpositional geometric properties. In general it is not easy to see how to separate positional from nonpositional geometric properties of arbitrary subsets of the volumes of the datum and the feeling. But in the particular case of strains the task is facilitated by the fact that these volumes are dominated by flat loci of points. Once the sets of corresponding flat loci are known, the nonpositional, or eternal, geometric properties of arbitrary subsets of the volume of the strain are likewise known. The eternal properties of any arbitrary locus of points of the volume of a strain are all of those geometric properties of the locus which are entailed by the flatness of the component loci.

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WHITEHEAD'S EXPLANATION OF THE PAST

UNDER this general heading I shall discuss two topics: (I) how Whitehead introduces the concept of pastness into his system, and (II) what explanation of the past his principles require him to give. The chief value of the discussion may be to raise certain questions, first about the way his system is constructed and second about the way it can be used.

I

By "pastness" I mean the relation "being in the past of," the converse of which is "being a successor of." X is in the past of A (A succeeds X) if X conditions A, if A does not condition X, and if their durations do not overlap. If X is in the past of A, then, when A exists, X has come to an end.

To see how Whitehead introduces this concept into his system, let us begin with the scheme of categories stated in *Process and Reality* (I, ch. II, secs. II and III). As Whitehead says, Part I is practically unintelligible by itself. But suppose we have studied the rest of the book, and his other writings, with care, and then return to the categoreal scheme. We shall understand better what

it says, but there is also a danger that we may read into it some things it does not say.

For example, having read many later passages where Whitehead points to experiences of causal efficacy and others where he explains transition from past to present in systematic terms, we are strongly tempted to suppose that the concept of pastness is a part of the categoreal scheme. Is this true? Clearly, the scheme does not rule out this concept. My question is: Does the scheme itself require this concept?

Whitehead does not claim to introduce all his systematic concepts in the categoreal scheme. On the contrary, he explicitly introduces concepts that the scheme permits but does not require, for example, the concept of God, the concept of the extensive continuum, and the concept of enduring objects. These he introduces in Chapter III under the title, "Some Derivative Notions." So these are systematic concepts but not categoreal concepts. They are required to interpret certain generic but contingent features of the world we experience.

This distinction is important for seeing what sorts of truth-claims are made in Whitehead's philosophy. We can distinguish categoreal statements (like "Every actual entity attains a satisfaction") which, in order to be true, must be required by the categoreal scheme, from other systematic statements (like "There is a nontemporal actual entity") which, in order to be true, must be permitted by the scheme.[1] Then we can ask: Is "Some actual entities are in the past of others" categoreally true?

Let us put our question into a larger setting by looking at some of the key concepts in Whitehead's theory of time, as follows:

1. Concrescence (and satisfaction)
2. Transition (inheritance, causal efficacy, influence)
3. Duration (contemporaneousness, unison of becoming)
4. Coordinate divisibility of regions (*qua* temporal quanta)

Let us ask whether all these are required by the categoreal scheme or whether, on the contrary, some are derivative notions. We can dispose of the clear cases first and then deal with the doubtful ones.

1. Concrescence is clearly one of the basic concepts in the categoreal scheme. A whole set of categories (the categoreal

[1] But not all statements permitted by the scheme are systematically true (e.g., "There is no extensive continuum"). His philosophy also includes many presystematic statements (like "We remember past events") and post-systematic statements (like "A stone is a nexus of actual occasions"), and these have to be judged in other ways. We shall come to these later.

obligations) is devoted to explaining the categoreal conditions a concrescence must satisfy. But we must remember Whitehead's often-repeated remark that the process of concrescence is not in "physical time."

2. It is equally clear that coordinate divisibility is not a categoreal concept but a derivative notion, belonging to the larger concept of the extensive continuum. Whitehead says plainly (PR 53) that extensive continuity is not a necessary feature of all possible cosmic epochs. This implies that it is not a categoreal concept.

3. The concept of contemporaneousness, on which the concept of a duration depends, is not expressed or explained in the categoreal scheme, not even in the category of transmutation (categoreal obligation vi) where it would be most likely to appear. Whether it is required by the scheme depends in part on what we discover about transition. For Whitehead's explanation of contemporaneousness depends on the concept of pastness, which belongs to the concept of transition.

4. Let us see, first, what passages in the statement of the scheme seem to require the more general notion of transition or some of its elements and, second, whether any of these require the more specific concept of pastness. I find five points that require attention, and beg the reader's patience through some exegesis: (*a*) a concrescence "originates from" an actual world (category of explanation v); (*b*) one actual entity is "objectified" in another (cats. exp. viii, xxiv); (*c*) the ontological principle can be termed "the principle of efficient, and final, causation" (cat. exp. xviii); (*d*) the subjective aim of an actual entity is at intensity of feeling in "the *relevant* future" as well as in the immediate subject (cat. ob. viii); and (*e*) "in each particular instance" the process of becoming (concrescence) "ceases" with the attainment of coherence (cat. exp. xxii).

Now, though *a*, *b*, and *c* introduce certain elements of the full concept of transition (e.g., "objectification"), they do not introduce the concept of pastness. The reason is that they do not require a distinction between past and contemporary actual entities. We are not told in *a* that the actual world of an actual occasion consists of only those actual entities in its past, though we know this from later developments. Nor is causal objectification distinguished from presentational objectification (i.e., of the contemporary world) in *b*. Similarly, *c* does not say or imply that there is no efficient causation between contemporaries, though we know this is a cardinal principle in Whitehead's cosmology.

So there is room to argue that the categoreal scheme does not of itself require that influence run from past to present. We could put this by saying that the scheme leaves open the question whether contemporary actual entities influence one another. But it would be much better to say that this question is not even raised, much less decided, because the concepts of pastness and contemporaneousness are not introduced.

Now consider passages *d* and *e*. Since *d* involves a distinction between present and future, we might think it required the concept of pastness. But the reference to a relevant future does not need to involve pastness. Its meaning could be satisfied by saying that some actual entities come into existence (or that they continue to exist) while others are in existence. It does not strictly imply that the coming into being of a relevant future involves the ending of the present subject. Nor does it deny this. My point is that the question is not raised.

The point of category of explanation xxii (*e*) is to explain the self-formation of an actual entity. This is done formally in the first sentence of the statement and more vividly in the second ("self-creative" for "self-formation"). The third sentence, beginning with "Thus," uses the explanation to elucidate "becoming" and, in its second clause, *adds* the remark about cessation. What is it that ceases? Clearly, the process of becoming (concrescence). But this is not the same as saying that the actual entity ceases. The specific point of the clause is not to say that an actual entity "perishes" but to say that every actual entity achieves a satisfaction. This is the "attainment" at which the process of self-formation is directed. Here as elsewhere the problem is to read the categoreal scheme for what it says and strictly implies, and not to read into it the systematic development that follows after derivative notions have been introduced.

Thus there are reasons for saying that the categoreal scheme does not introduce the concept of pastness either explicitly, by using and explaining it, or implicitly, by requiring it. This conclusion can be put by using the following statements:

P. Every actual entity succeeds some other actual entity.
Q. Some actual entities succeed other actual entities.
R. No actual entity succeeds another actual entity.

Then *P*, *Q*, and *R* are all categoreally permitted; *Q* is systematically true; and *P* and *R* are systematically untrue. *P* is systematically untrue because Whitehead introduces, as a derivative notion, the

concept of an actual entity (God) that does not succeed [2] any other actual entity. God is "primordial." Q is systematically true because Whitehead introduces the concept of temporal actual entities (i.e., actual occasions) which, if the concept of a non-temporal actual entity is derivative, must be a derivative notion also. And if Q is systematically true, R is systematically untrue.

If this conclusion is justified—and I do not suppose I have proved it, but I hope I have shown how it is arguable—it has two important consequences.

1. The categoreal scheme would permit the possibility of a universe (or a "cosmic epoch") in which the relation "being in the past of" was not instantiated. All actual entities would be conditioned by others, as some A by some X. But since X would not come to an end, X would not be unambiguously in the past of A. For all we are told in the categoreal scheme, a universe like this is abstractly conceivable. It happens that our world is not like this, for in our world the relation "being in the past of" is abundantly instantiated. It is clearly, but contingently, true that in our world some things are unambiguously in the past of others. Hence if the categoreal scheme is to apply to this world, there must be some actual entities that come to an end. Whitehead says:

> In the temporal world, *it is the empirical fact* that process entails loss: the past is present under an abstraction. But there is no reason, of any ultimate metaphysical generality, why this should be the whole story (PR 517, my italics).

2. The categoreal scheme would permit (though it does not require) the conception of an actual entity that is not unambiguously in the past of any other actual entity (since it never comes to an end) and does not succeed any other actual entity (since it has no beginning).

II

The second question I wish to raise is this: What sort of explanation of the past do Whitehead's principles of explanation require? With all due respect, this is not the same as asking what Whitehead thought they required.

Whitehead's conception of speculative philosophy requires three sorts of statements involving references to the past. I shall call these (1) *descriptions*, (2) *interpretations*, and (3) *explanations*.

1. First he must assemble the main facts about our experience of the past, drawing on our ordinary experience, on science, and

[2] In the sense defined at the beginning of this section. Of course, all other actual entities affect God.

on the reflections of earlier philosophers, putting these facts into descriptive statements. So he describes our sense of the efficacy of the immediate past, acts of remembering, and our practical reliance on causal influence, in what might be called *phenomenological* or *presystematic* statements. He remarks, among other facts, on the "roughness" of our experience of time. Instead of a steady stream of time, smoothly flowing, there are discrete happenings. Events occur after other events. This is why a systematic concept of pastness will be needed.

2. Then, in *postsystematic* statements, he must interpret the facts he has described. For example, the following would be an interpretation of some particular experience of efficacy: "In this experience the subject is directly prehending some immediately past actual occasions." In statements like this both nonsystematic terms (e.g., "experience") and systematic terms ("prehend," "actual occasion") occur. The nonsystematic terms are taken over from ordinary language, from science, from traditional philosophical usage, or from other sources.

3. The systematic terms require explanation in *systematic* statements, telling for example just how an actual occasion prehends an immediately past occasion. Whitehead must explain how, in objectification, an objective datum is derived from an initial datum by abstraction, involving negative prehensions, and how subjective forms of feeling are repeated.

Two points about interpretations (postsystematic statements) should be noticed in passing. First, they are not immediately deducible from descriptions, because their systematic terms express constructed concepts, not (we might say) natural concepts (including here those scientific and traditional philosophical concepts which have become, so to speak, naturalized). The meaning of these terms is given in the categoreal scheme and in later systematic developments. So the truth of interpretations depends not only on the truth of the descriptions but also on the truth-claim of the system as a whole. The second point is that, if the conclusion of section I (above) is justified, interpretations of the past cannot be explained by reference to the categoreal scheme alone. This is because the categoreal scheme does not tell us that for some actual entity A, some of the actual entities objectified in A are in A's past. So derivative notions as well as categoreal concepts are involved in the explanation.

One of Whitehead's principles of explanation is the "ontological principle" (cat. exp. xviii). How does this principle apply to the explanation of the past? It is a rule governing systematic

explanations and might be put in the following way: When you mention some condition to which a particular process of becoming conforms, you must refer to some actual entity (or entities) as the reason for the condition. So the principle governs systematic explanation of all the conditions of a concrescence, including conditions from the past, if any. (Incidentally, the principle clearly rules out creativity as an ontological ground. It calls for actual entities, and creativity is not an actual entity, nor indeed an entity of any other systematic kind.)

Now, when the concept of pastness is introduced into Whitehead's system, does this put an additional strain on the ontological principle? Are we required to explain, under the ontological principle, not only why a condition of a concrescence has the character it has but also how it comes to be an effective condition *now*?

The reason that this is at least a specious problem, if not a real one, is as follows. Whitehead undertakes to describe and interpret the discontinuity of experience in time. Some things happen after other things have stopped happening. In this way the world is "rough" in its temporal dimension as well as spatially. It is not the smooth world of classical mechanics.[3] To interpret this feature of experience Whitehead introduces the concept of temporal actual entities (actual occasions), which can be serially ordered in such a way that some are in the past of others. Conversely, some succeed others. This makes possible a systematic explanation of the epochal character of time.

Now, if some actual occasion X is succeeded by some actual occasion A, it is clear that X can be the reason (under the ontological principle) for *what* is given for A. But if A happens after X, it is not so clear that X can be the reason why this is in fact given for A. So the question arises whether the ontological principle properly applies to the givenness of the past as well as to the character of the data from the past. And if it applies, what systematic explanations of givenness are possible?

All I propose to do about this puzzle is to set out briefly three arguable views on the matter, though I should add that the arguments for the first two views are stronger than I used to think.

1. One view is that the ontological principle does not properly apply to the givenness of the past. What is to be explained under the principle is not the fact that a concrescence conforms to conditions, but only the conditions to which it conforms. The concept

[3] Of course Whitehead also undertakes to explain the smoothness of the world of classical mechanics. That world is an abstraction from the world of concrete experience, and he tries to show how that abstraction is made.

of conformity to some conditions or other is built into the categoreal scheme to interpret a pervasive feature of experience. So the fact of conformity as such, in any particular case whatever, does not call for any special systematic explanation under the ontological principle. Now if conformity as such does not require special explanation, then givenness as such does not require special explanation, for the concepts of conformity and of givenness are tightly linked together. What is to be explained under the ontological principle, in a particular case, is not that something is given but only what is given. On this view, the problem about a ground of the givenness of the past is only a specious problem, not a real one.

The main strength of this view is its simplicity. Also, the fact that Whitehead does not clearly mention the problem in his writings is a reason for attributing this view to him. But our question is: what do his principles require?

2. A second view is only a little less simple. This is that the ontological principle properly applies to the givenness of data from the past as well as to the character of the data and that the objectified actual occasions are the reasons for their own givenness for the present.

The natural (but not conclusive) objection to both these views is that they are too simple. They leave us with a mystery on our hands, namely, the mystery of how something past can be effective in the present. We may say that this is simply part of the general mystery of time. But when we are interpreting Whitehead, our problem is not whether we are able to admit such mysteries but whether Whitehead can admit them. Do his principles permit him to leave this mystery on our hands? Perhaps they do. Perhaps, indeed, they require him to do so, but I am not sure this is the case.

One way this mystery could be avoided is by saying that there really are no endings. Things live on and on, as real as ever they were. So we would have no good use for the concept of pastness. But Whitehead needs and uses this concept. Another way to avoid the mystery is to say that there really are no beginnings. What seems like a birth is really a sleep and a forgetting. But I suppose it is clear that Whitehead thinks there are real and novel beginnings. The mystery appears because Whitehead wants his speculative philosophy to do justice both to real endings and to real beginnings.

3. A third arguable view is that the ontological principle applies to the givenness of the past and that God is the ground of the givenness of the past. There are at least three objections to this view, as follows:

(*a*) Whitehead does not clearly assign this function to God.

(*b*) The explanatory mechanism we would have to construct[4] is bound to seem somewhat artificial. (However, if pastness is a derivative concept, some categoreal difficulties would be mitigated.)

(*c*) This would leave us liable for an explanation of the ground of givenness of the data of God's physical prehensions. (But since no actual occasion is in the past of God—nothing happens before God—this is not the same problem as in the case of actual occasions. Here, applying the ontological principle would not mean asking for a ground of the givenness of the *past*.)

As I have meant to suggest, these objections are not conclusive, though they have some weight. But suppose we should reject view 3 for these or other reasons. Then we should still have to recognize two functions of God in relating a present actual occasion (as A) to a past occasion (as X). First, God makes it *possible* for A to absorb some abstraction from X into its own satisfaction, in the following way. A can be positively influenced by X because A's subjective aim permits this. Now A's initial conceptual aim is derived from a hybrid physical prehension of God. And the ordering of pure possibilities (eternal objects) that conditions this hybrid prehension is determined in part by A's past, including X. But this implies a constant aim in God's experience, something determinable but constant in God for X to determine. In this way God's constant subjective aim at maximum intensity is a reason (under the ontological principle) for the possibility of A's having positive prehensions of X. Also, X is a reason for the particular pattern of divine experience which orders the relevance of possibilities for A's subjective aim.

Second, God makes it *necessary* for A to conform to the past, in the following way. A's subjective aim, inherited from God (but with modifications), is such that it can be achieved only if A conforms to X in some way. A macroscopic example: If I am ill and want to be well, I must treat the causes of the illness. Only by taking account of the past can the aim of a finite concrescence be achieved. In this way conformity to God requires conformity to the past also.

Neither of these functions nor both together amount to quite the same thing as being the ground of the givenness of the past. But they make God an essential condition—though not the only condition, of course—of the effectiveness of the past in the present,

[4] See *An Interpretation of Whitehead's Metaphysics* (New Haven: Yale University Press, 1959), pp. 327–330.

and are in this way part of Whitehead's systematic explanation of the past. They help to give an interpretation, in systematic terms, of the following: "We owe to the sense of Deity the obviousness of the many actualities of the world . . ." (MT 140).

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THE VISION OF BEAUTY AND THE TEMPORALITY OF DEITY IN WHITEHEAD'S PHILOSOPHY

Religion will not regain its old power until it can face change in the same spirit as does science.

WHITEHEAD once said that Aristotle's metaphysical deity was hardly "available for religious purposes" (SMW 249). Seventeen years later Stephen Ely came to the conclusion that Whitehead's God was defective in the same respect.[1] The presuppositions of what sort of a God *would* be available for religious purposes emerge slowly in Ely's essay. Whitehead's God is then shown as having failed to meet the demands imposed by these presuppositions. It is my theme that Whitehead was attempting—among other things—to develop a notion of God that might select the best in religious consciousness, rather than merely conform to prevailing notions of what kind of God might be satisfactory.

To perform such a task in a few pages means simplification of perhaps the most complex feature of Whitehead's mature philosophy. I sympathize in advance with those whose knowledge of Whitehead may be offended by a skeletonizing of his doctrine of God.

But if we must be simple, we can also be systematic. The first thing to notice is that at least Whitehead is not blown up by his own petard. Whitehead's statement that Aristotle's God was not available for religious purposes meant only that the Prime Mover appeared simply as a system-completing necessity and that there was about the awareness of the Prime Mover none of that warmth of feeling which characterizes the religious attitude. It should be clear to all critics, friendly or not, that this absence of feeling in the relation between man and God is precisely what Whitehead purposes to remedy. His task can thus be stated simply as the effort to close the gap between a God of thought and a God of feeling. Such a procedure was not a forced procedure for White-

[1] Stephen Ely, *The Religious Availability of Whitehead's God* (Madison: University of Wisconsin Press, 1942).

head, because feelings, in an extended sense of the word "feeling," are the elementary relations in the world process. The idea of a felt God, Himself capable of feeling and therefore a God not wholly remote or intellectually defined, is Whitehead's correction of Aristotle's barren conception of a Prime Mover.

With this initial point out of the way, we may proceed to a direct examination of Ely's criticism, ending with a statement of an important but not commonly remarked feature of Whitehead's philosophy of religion. In this essay I propose to show the following: (1) that Ely improperly solicits support for his view from Plato, whereas Whitehead actually derives much from Plato; (2) that Ely seems to mean by "religious availability" something that has but little support from religion; (3) that Whitehead's repeated recourse to aesthetic vocabulary is essential in his relating of the temporality of God to the overcoming of evil; (4) that Whitehead's metaphysics is not simply a metaphysics including religious doctrine and thought, but that it is a metaphysics of religious experience as well. (5) Fifthly, I shall try to summarize the problem of "religious availability" in the light of point 4; and, finally, (6) I append some remarks on the usefulness of aesthetic language.

I

We shall begin with a sustained passage from Ely:

> All values are then fundamentally aesthetic. This makes it easier to understand why God, overwhelmed with an immense vision of cosmic beauty, is not concerned with our finite sufferings, difficulties, and triumphs—except as material for aesthetic delight. God, we must say definitely, is not primarily good. He does not will the good; He wills the beautiful. Suffering humanity might well call this God's attention to the utterance, made near the close of his life, of a deeply religious man. "When a man honours beauty above goodness," said Plato, "this is nothing else than a literal and total dishonouring of the soul" (Ely 52).

This passage is a tangle of many confusions. Since Whitehead is substantially influenced by Plato and Plato is here so badly represented, we must at the outset unscramble the confusion on Plato.

First, it is hard to explain why Ely uses only a fraction of a sentence from the Bury translation of the *Laws,* to make his point. The full sentence reads as follows:

> Again, when a man honours beauty above goodness, this is nothing else than a literal and total dishonouring of the soul; for such a statement asserts that the body is more honourable than the soul,—but falsely, since nothing earth-born is more honourable than the things of heaven, and he that surmises other-

wise concerning the soul knows not that in it he possesses, and neglects, a thing most admirable.[2]

Thus Plato is speaking here of sensuous beauty, not the beauty that appeals to soul or spirit. This latter idea of beauty, far from being condemned by Plato, is exalted in the highest terms.

Plato uses two words that are both translated as "beauty": *τό κάλλος* and *τὸ καλόν*. The double-lambda word has the sense of adornment or *décor*, sensuous beauty. The single-lambda form has the sense of nobility or worth. Thus, for instance, the word *καλοδιδάσκαλος* means "a teacher of virtue." In the famous "ladder of beauty" passage at the end of Diotima's oration in the *Symposium*, Plato uses the single-lambda form exclusively.[3] *The Republic* appears not to use the double-lambda form at all.[4] So both in context and by common usage, the cited passage has nothing to do with divine beauty.

Secondly and briefly, the word that Bury translates as "goodness" is *ἀρετή*, usually translated as "virtue." It is more happily rendered as "excellence," but in no case should it be confused with moral goodness, that is, with *ἀγαθόν*. Plato's distinction is that between the active striving to excel and the passive enjoyment of the sensuously beautiful. Plato makes no effort to raise the morally good above the spiritually beautiful.

On the contrary and thirdly, if it be important to know what Plato thought of genuine beauty, we may go to the famous passage in the *Symposium*. Here Plato says:

'. . . so that in the end he comes to know the very essence of beauty. In that state of life above all others, my dear Socrates,' said the Mantinean woman, 'a man finds it truly worth while to live, as he contemplates essential beauty (211C–D; p. 207).'

Commenting on this passage, F. M. Cornford says of beauty that for Plato it simply *is* the good, considered as an object of desire (Cornford 216).

I have labored this point because it bears on Whitehead's approach. Plato says in the passage just quoted that a man finds it worthwhile to live in the contemplation of essential beauty. However, this contemplation is not a passive reverie. Rather, it is a disciplinary vision: disciplinary in that it subordinates other values to itself, and a vision in that it is pervasively present as the ulti-

[2] Plato, *Laws*, tr. by R. G. Bury (London: Heinemann, 1926; Loeb Classical Library), 727 D–E, vol. I, p. 325.

[3] Plato, *Symposium*, tr. by W. R. M. Lamb (London: Heinemann, 1939; Loeb Classical Library).

[4] F. M. Cornford, *The Republic of Plato* (Oxford: Clarendon Press, 1941).

mate in objective worth. The same vision is called by a moral name in *The Republic*.

II

Let us put aside any support from Plato. There is little else in Ely to show that Whitehead's God is not religiously available. Ely's complaint takes the curious form of arguing that if what I regard as good is not instrumented by God, then such a God is not religiously available. For instance, he says,

> God can work wonders, but I do not think he can merge me and my values into an indefinitely immense system and still claim that I have maintained *my* individuality and my values. Of course, if human values are not fundamentally the same as God's values, they will not be preserved in any case. Then the whole question falls, as there is in that case no possibility of showing that God is good in our sense of the term (Ely 49).

Ely's charge is "guilty of having failed to take account of human values, having failed to preserve individuality, and having failed to be good in our sense of the term." Now this is surely an odd charge to bring into court. For millions of men who certainly have felt that God was religiously available, it is never the case that one postulates the human good as the norm and then asks whether or not God conforms. Quite the opposite. The good was understood by definition to be that which God wills. Job's God was available for religious purposes. The Psalmist sings, "How unsearchable are his judgments and his ways past finding out." A God who punishes all mankind for the sin of the first man, which sin he permitted with foreknowledge, a God who deals out retribution, who exacts penalties, who gets exceeding wroth if his people worship his ethnic first cousins, who crushes whole nations in his anger (what of the nursling child and the frightened, unknowing mother? They are his victims, too)—all these have been available for religious purposes. Having little concern for the human conception of value does not seem to be a serious limitation on God's religious availability. In some cases, it almost seems to be an absolute prerequisite for divine status.

Unless pre-Danielic Israel had no religion, we should not take this line of argument very seriously. In fact, if we begin looking about for Gods that will meet this demand of preserving *my* individuality and *my* values, we find few of the great religions that will supply us. In Hinduism and Buddhism my individuality is metaphysically denied. In Islam and late Judaism my individuality is problematically capable of survival, but *my* conception of value does not have much religious force unless it conforms to

God's will. Only in certain strains and aspects of Christianity do we find this demand *both* for a personal God who cares for my values and for a personal immortality for my individuality.

III

We are forced to the conclusion that Ely's complaint is really a normative judgment, not a descriptive one: Whitehead's God is transmoral, a divine artist, some of whose pigments are genuine evil and genuine suffering. Ely deals with two kinds of evil: the evil of suffering (Ely 44–45) and the evil of perishing (Ely 48). In *Process and Reality* Whitehead says,

> . . . the immediacy of sorrow and pain is transformed into an element of triumph. This is the notion of redemption through suffering, which haunts the world. It is the generalization of its very minor exemplification as the aesthetic value of discords in art (PR 531).

Of this view Ely complains that Whitehead supposes that "playing a wider role will make one content with his lot" and that "we can find, at best, only a vicarious joy in our sufferings and our tragedies. The actual experience of joy is God's, and we can rejoice only because God has it" (Ely 49–50). To make matters worse, Whitehead says in *Adventures of Ideas* that "The real world is good when it is beautiful. . . . Thus Beauty is left as the one aim which by its very nature is self-justifying" (AI 345, 342). This passage leads Ely to the misuse of Plato cited at the outset of the present essay.

Before we can go further we must very briefly review Whitehead's conception of God in the mature period. God is there presented as having two natures, which he calls the Primordial Nature and the Consequent Nature. The Primordial Nature is the repositum of all possible value, but only as possible. In this repositum there lies the entire multiplicity of eternal objects, which are all the qualities, characteristics, or properties that could characterize any event or set of events. There is no single final ordering of these eternal properties, but the possible orderings of them are themselves complex eternal objects (SMW ch. X) and are therefore part of this multiplicity.

God as primordial is mental in that the concepts of all possibilities lie in Him, only ideally, not actually (PR 521–522). This realm of possibility is not merely a celestial parts department or emporium. It is the realm to which the event-in-process-of-formation is drawn as it proceeds from its fixed background of fact. The primordial nature of God is like Aristotle's Prime Mover in that it is eternal, complete, and the object of desire toward which

all things are drawn. But it is not conscious, for consciousness requires the fusion of conceptual *and* physical feeling. Clearly, this aspect of God is *not* available for religious purposes. The entire issue thus rests with the Consequent Nature of God. Where the Primordial Nature is complete, the Consequent Nature is incomplete. Where the Primordial Nature is nonconscious, the Consequent Nature is conscious. Where the Primordial Nature proffers possible values, the Consequent Nature conserves actual values. Where the Primordial Nature is eternal, the Consequent Nature is everlasting (PR 521–524).

The everlastingness of God is prefigured by Whitehead in *Religion in the Making,* where he says of God that He is the "nontemporal actual entity," what men "call God—the supreme God of rationalized religion" (RM 90). Two terms stand out: the *non*temporality of God, and rationali*zed* religion. The nontemporality of God, as Whitehead abundantly explains in the closing passages of *Process and Reality,* is not His eternality; this is the property of the Primordial Nature. What is eternal is nonactual; i.e., it is not itself time-structured or time-dependent in order to be what it is. The Consequent Nature of God is derived "from the temporal world": it is what is "permanent," in which the temporal world is "perfected" (PR 527). But neither God nor the world reaches "static completion" (PR 529); this static completeness belongs to the Primordial Nature. The incompleteness of the everlasting nature of God rests on the fact that time is real, and the Consequent Nature of God constantly receives the datum of completed actual entities into the unending completion of His Consequent Nature. "The purpose of God is the attainment of value in the temporal world," says Whitehead (RM 100).

However, this "nontemporality" of God needs explication. *Three distinct modes of existence with respect to time are presented:* (1) The Primordial Nature of God and the eternal objects therein are *time-free,* eternal. (2) Actual occasions are *time-structured;* they and all the societies they compose, except the world as a whole, come to completion—i.e., to temporal closure—and thus become a challenge to the Consequent Nature of God, who must both conserve them in their objective immortality and compose them in that harmony which is His constant aim. (3) The third type of existence is not time-structured, but it is *time-concerned,* so to speak, and *time-dependent;* this is the Consequent Nature of God, abiding, everlasting, and in this sense temporal and incomplete.

The objects of God's will are the disparate actualities, which, when complete, slip backward into the stream of time. They replace one another. What does not change is the will to harmonize

them. The will is unchanged by time, everlastingly the same, yet always engaged in the struggle with what is temporally delivered to it. What these temporal entities become is in some sense free for them and irrevocable, insofar as they are individual. But their freedom and their completedness pose a problem beyond themselves which, by reason of their completedness, they are impotent to deal with.

With this brief statement of the role of the Consequent Nature of God we must be here content, though it offers many puzzles. What I have tried to show is that Whitehead renders rational a concept from the familiar language of Christianity which might otherwise be challenged as to its rationality—the eternal will of God. Where Christianity writes "eternal," Whitehead writes "everlasting." Where Christianity writes "will," Whitehead writes "aim."

IV

The will of God to produce harmony from what is partly evil, while granting each creature its freedom to fail and to suffer, is more commonly explained in aesthetic language. But since it is a will and since it proposes to mitigate what is evil and since Whitehead repeatedly says that God "saves" the world (e.g., PR 525, 526) through this harmonizing, we can hardly say that Whitehead's God has no moral stature nor any moral goal. Moreover, Whitehead says, "Morality of outlook is inseparably conjoined with generality of outlook" (PR 23). He also says, in an infrequently remarked passage which links morality with art, "The canons of art are merely the expression, in specialized forms, of the requisites for depth of experience. The principles of morality are allied to the canons of art, in that they also express, in another connection, the same requisites" (PR 483). To the question of why the language of morality occasionally suffers in Whitehead's hands, the answer seems clear: it is the complacency and self-interestedness of much of what is called moral that he wishes to avoid. Thus he says, "Of course it is true that the defence of morals is the battle-cry which best rallies stupidity against change. Perhaps countless ages ago respectable amoebae refused to migrate from ocean to dry land—refusing in defence of morals" (AI 345–346).

The temporality of God leads us to a highly significant phrase: "the Supreme God of rationalized religion." Notice that Whitehead does not say "rational religion." What is it that is *rationalized,* that is, placed in rational coherence with the rest of experience? The answer is: religious experience. It is false to think that some kinds of experience come with special warrants of ac-

ceptability that other experience does not have. But disparate elements of experience must be rendered coherent. The organization of Whitehead's work makes this abundantly clear.

1. The section from which this phrase is taken is called "A metaphysical description." Whitehead says of it, "The . . . following description is set out for immediate comparison with the deliverances of religious experience" (RM 89).

2. The section before this one begins with the sentence, "In the previous lectures religious experience was considered as a fact" (RM 86).

3. The previous lecture is nothing less than a review of "The Religious Consciousness in History," "The Description of Religious Experience," "God," and "The Quest of God." These are not my titles; they are Whitehead's. Whitehead thus takes up where William James's Gifford lectures leave off. James was concerned with the phenomenology of religious experience. Whitehead explains the metaphysical accommodation of *religious* experience, which is a factor within the whole of experience. The religious availability of Whitehead's God thus appears in a quite different light.

Let us now skeletonize the complaints about this availability and reinforce them. My purposes and my conception of the morally or naturally good are not met in the world about me. They fail in many ways. I am faced with "the evil of perishing." Not only do I perish, but, before my time, those whom I love perish. Whitehead's son Eric was shot down in the First World War. Whitehead dedicated his first work in the philosophy of science to his dead son. I reproduce the dedication, not for its sentiment but for its doctrinal content.

To Eric Alfred Whitehead, Royal Flying Corps, November 27, 1898 to March 13, 1918. Killed in action over the Forêt de Gobain giving himself that the city of his vision may not perish. The music of his life was without discord, perfect in its beauty (*Principles of Natural Knowledge*).

The key terms here are "vision," "not perish," "beauty." The compact thought is existentially significant. Beyond the horror of that death and his own sorrow Whitehead conceived his son's vision in the language of beauty. So much for what philosophers call natural evil. "Your character," Whitehead says, "is developed according to your faith" (RM 15).

Again, my moral vision is constantly abused. The wicked prosper. The good suffer, and my assuagement for these facts is the aesthetic vision of the Consequent Nature of God, forever com-

pleting His own existence by a harmony that rescinds the objective evil but gives me, as finite, no mercy. Cold comfort, Ely says. "It is not likely to give anyone much comfort to know that no matter what happens in this world, God can see it in an ideal setting that makes it an enjoyable sight" (Ely 41).

Whitehead is not unaware of this issue. He says, in one of the most stinging passages in the entire corpus,

> Good people of narrow sympathies are apt to be unfeeling and unprogressive, enjoying their egotistical goodness. Their case, on a higher level, is analogous to that of the man completely degraded to a hog. They have reached a state of stable goodness, so far as their own interior life is concerned. This type of moral correctitude is, on a larger view, so like evil that the distinction is trivial.
>
> Thus if God be an actual entity which enters into every creative phase and yet is above change, He must be exempt from internal inconsistency which is the note of evil. Since God is actual, He must include in himself a synthesis of the total universe. There is, therefore, in God's nature the aspect of the realm of forms as qualified by the world, and the aspect of the world as qualified by the forms (RM 98).

So much then for moral evil as well as natural evil.

Whitehead's intention is not primarily to render the idea of God palatable to anyone's idea of religious need and to accomplish this on metaphysical grounds. This is not a buyer's market, where the metaphysics must either put up an acceptable product or withdraw it when it is found not to be religiously available. The whole project is of an entirely different sort: to explain the relation of religious experience to experience generally. What is true of experience generally is true of experience in its religious aspect, Whitehead says.

> The moment of religious consciousness starts from self-valuation, but it broadens into the concept of the world as a realm of adjusted values, mutually intensifying or mutually destructive. The intuition into the actual world gives a particular definite content to the bare notion of a principle determining the grading of values. It also exhibits emotions, purposes, and physical conditions, as subservient factors in the emergence of value (RM 59–60).

A metaphysics of religious experience is confronted with certain *facts*. These facts are the ability of the subjective purposes in consciousness to rise above their common limits, transpersonally or even transmorally—a point heavily emphasized by Kierkegaard, incidentally—to a grasp of value that erases temporal losses. Nor will it do to attribute this transcending ability and the religious experience that fulfills it to pathology, for it carries with it not only private authenticity but publicly noticeable beatification. We may play such idle games as are commonly played by David Hume and Bertrand Russell, in which skeptical theory is pursued with

endless diligence, but then practical success becomes a mystery, a kind of secular miracle. There is, on the one hand, the subjective persuasion of religious experience, which stretches beyond standard conceptions of value. On the other hand, there is the worth of lives so guided. The correlation of these facts is badly in need of understanding. To use Whitehead's term, they need to be *rationalized*. Some men do have such vision beyond average values, and it has publicly noticeable consequences. Any metaphysic worthy of the name must accommodate these facts. The primary role of a metaphysic is to describe what is, systematically and rationally. Derivatively it may lead men to deeper insight.

We may say then of the question of the religious availability of Whitehead's God that it is improper, provided some ill-defined standard of availability is pursued at the outset. "O God . . . ," says W. H. Auden in a mocking poem, "Leave Thy heavens and come down to our earth of waterclocks and hedges. Become our uncle. Look after Baby, amuse Grandfather, escort Madam to the Opera, help Willy with his home-work, introduce Muriel to a handsome naval officer."[5] Some men have got beyond such a conception of God and found him to be religiously available. As things now stand, the availability is admittedly not widespread. For that matter, neither is the fact that the sun doesn't move through the heavens, although this is much easier to demonstrate. We are not yet done with the Ptolemaic period, when man and his earth were at the center of things.

V

One final point remains. Aside from the misleading character of moral language, why should the aesthetic metaphor be used? Or better, if some vocabulary must be given an extended meaning, why the aesthetic vocabulary?

No doubt every kind of value terminology is somewhat limited when it comes to the problems of religion. Perhaps the language of religious value should be distinct from all other forms. Yet religious value and the language of religious value are extremely sophisticated. But by what means shall the secular mind be brought into religious sensitivity? Or by what series of steps, starting where, does a young mind, aesthetically alive but perhaps quite unresponsive to the language of morals or the language of mysticism, come to any religious understanding? It is a failure of much religious practice that it confines itself to the language and thought of morality, which tends to be restrictive, and the language

[5] W. H. Auden, "The Massacre of the Innocents," in *For the Time Being* (New York: Random House, 1944), p. 122.

and thought of mysticism, which is supernatural. The total confinement of religious insight to what is on the one hand antinatural or on the other hand supernatural does not seem like a very promising procedure for either engendering or developing religious sensibility. If organized religion finds itself limited in its capacity to appeal to people of intellect or sensibility, it is not man who has rejected God, but rather peddlers of the image of God who have rejected man. Aesthetic sensibility is the most natural of all types of value sensibility. It begins with the delight of the child in the world of the senses. It is often the substitute for moral self-restraint in adolescents who may have come privately to the conviction that moral restriction and moral taboo have no claim upon their own decisions. Commitments to taste, discrimination, and personal fastidiousness may step forward and produce a behavior masking as moral but actually based upon aesthetic self-respect. At the highest level of religious achievement, that of mystical insight or mystical commitment, the common literary representation is always in terms of beauty. From beginning to end it would seem that, if some metaphor must be chosen for religious insight and for the relation of God to the world, the aesthetic metaphor is not only as good as any but perhaps better than any. I have tried to show briefly that it displays a deep and *continuous* current of value sensibility. In addition, where morality is restrictive, aesthetic sensibility and the aesthetic metaphor are creative. Where mere morality obtains, only the forbidden and the forbidding or else the compulsive and barren sense of duty operate. Over against this, the aesthetic metaphor and the aesthetic mode offer excitement, allurement, inspiration.

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THE ALGEBRA OF LOGIC AND THE THEORY OF DEDUCTION

WHITEHEAD'S earliest love was algebra, and it may be by way of algebra, the so-called *algebra of logic* in particular, that he came to logic. It is thus surprising to find *Principia Mathematica* so reticent on the twinship between the algebra of logic and the theory of deduction, or, as we might put it today, the (Boolean) class calculus and the propositional calculus. Whitehead and Russell note that "the properties of negation, addition, multiplication and inclusion [for classes] . . . are, with certain exceptions, analogous to the properties of negation, addition, multiplica-

tion and implication for propositions" (PMI 89). This, however, is about as far as they carry the matter. I should accordingly like in this commemorative issue on Whitehead to study two ways of converting postulates for the (Boolean) class calculus (CC, for short) into full-fledged postulates for the propositional calculus (PC, for short) and thus elaborate somewhat on the above-quoted remark.

I

That the properties of negation, addition, multiplication, and inclusion for classes are *analogous* to the properties of negation, addition, multiplication, and implication for propositions can be dramatically enough illustrated. Let the primitive signs of CC be a list of variables, the negation (or, better, complement) sign '$-$', the addition (or, better, union) sign '$\cup$', the identity predicate '$=$', and the two parentheses '(' and ')'; let the metaaxioms of CC be as in Huntington's 1933 paper:[1]

CC1: $(\alpha \cup \beta) = (\beta \cup \alpha)$,
CC2: $((\alpha \cup \beta) \cup \gamma) = (\alpha \cup (\beta \cup \gamma))$,
CC3: $(-(-\alpha \cup -\beta) \cup -(-\alpha \cup \beta)) = \alpha$,
CC4: $\alpha = \alpha$;

and let the rules of inference of CC be:

CC5: From $\alpha = \beta$ to infer $\beta = \alpha$,
CC6: From $\alpha = \beta$ and $\beta = \gamma$ to infer $\alpha = \gamma$,
CC7: From $\alpha = \alpha'$ to infer $\beta = \beta'$, where β' is like β except for containing occurrences of α' at one or more places where β contains occurrences of α.

The results (call them PC1–PC4) of substituting '$\sim$' for '$-$', '$\vee$' for '$\cup$', and '$\equiv$' for '$=$' in CC1–CC4 will all be tautologies and, hence, fit to appear among the axioms or theorems of PC; the results (call them PC5–PC7) of performing the same substitutions in CC5–CC7 will, on the other hand, preserve tautologies and, hence, be fit to appear among the primitive or derived rules of inference of PC.

That the properties of negation, addition, multiplication, and inclusion for classes are, *with certain exceptions*, analogous to the properties of negation, addition, multiplication, and implication for propositions can likewise be dramatically illustrated. Whereas CC1–CC7 yield as theorems all the valid formulas of CC, PC1–PC7 (in which, by the way, '$\sim$' and '$\vee$' may be treated as primitive

[1] See *Transactions of the American Mathematical Society*, **35** (1933), 274–304. CC1–CC3 appear in the paper as Postulates 4.3, 4.4, and 4.6 on page 280, CC4–CC7 as Postulates A, B, C, and D on the same page.

connectives and '$\equiv$' as a defined one[2]) yield as theorems only such tautologies as are of the form $\alpha \equiv \beta$.

Whitehead and Russell nowhere suggest what to add, by way of metaaxiom or rule of inference, to the propositional rewrites of postulates for CC if all tautologies are to be forthcoming as theorems. The problem, to my knowledge, was first tackled by C. I. Lewis in *A Survey of Symbolic Logic*[3] and in *Symbolic Logic*.[4] In the first passage Lewis suggests that a further axiom, or, as I would have it here, metaaxiom, be enlisted, namely: $\alpha = (\alpha = 1)$, where '$=$' must clearly be understood as '$\equiv$' and '1' as a propositional rewrite of the constant '1' of CC if the formulas collectively referred to by means of $\alpha = (\alpha = 1)$ are to be formulas of PC. The suggestion, however, is idle, since the formulas in question are already provable as theorems of PC by means of PC1–PC7. In the second passage Lewis makes a similar and equally idle suggestion, $(\alpha = 0) \equiv \sim \alpha$ doing duty this time for $\alpha = (\alpha = 1)$. Before laying down this new metaaxiom, Lewis notes, however, that $\alpha = 0$ and $\sim\alpha$, on the one hand, and $\alpha = 1$ and α, on the other, are henceforth to be interchangeable. This offhand remark of his constitutes a first solution to the problem, for PC1–PC7 plus the following rule of inference:

PC8: From $\alpha \equiv (\alpha \equiv \alpha)$ to infer α,

yield all tautologies as theorems. The fact may first have been mentioned by Tarski in the course of his Paris lectures of 1955.[5] A second solution to the problem is offered in another paper of mine,[6] where PC1–PC7 plus the following rule of inference:

PC9: From α and $\alpha \equiv \beta$ to infer β,

are also shown to yield all tautologies as theorems. Other rules of inference likewise fill the bill:

PC10: From $\alpha \equiv (\beta \equiv \beta)$, where β is any formula of PC, to infer α,

PC11: From $\alpha \equiv \beta$, where β is any one of PC1–PC4, to infer α,

PC12: From $\alpha \equiv (\alpha \,\&\, \beta)$ to infer $\alpha \supset \beta$,

PC13: From $\beta \equiv (\beta \vee \alpha)$ to infer $\alpha \supset \beta$,

the first two of which are essentially due to Tarski.

[2] To be more explicit, $\alpha \supset \beta$ may—as in *Principia Mathematica*—be short for $\sim\alpha \vee \beta$, $\alpha \,\&\, \beta$ short for $\sim(\sim\alpha \vee \sim\beta)$, and $\alpha \equiv \beta$ short for $(\alpha \supset \beta) \,\&\, (\beta \supset \alpha)$ or $\sim(\sim(\sim\alpha \vee \beta) \vee \sim(\sim\beta \vee \alpha))$.

[3] (Berkeley: University of California Press, 1918), pp. 222–224.

[4] Lewis and Langford (New York: Century Company, 1932), pp. 80–83.

[5] I owe this information to Professor Jean Porte. The lectures in question have not been published.

[6] The paper, written shortly before this one, is to appear in *Mind* under the title, "Boolean Algebra and the Propositional Calculus."

That PC1–PC7 and PC8 yield all tautologies as theorems can be demonstrated as follows. Let α be any tautology and let α^* be the formula of CC of which α is the propositional rewrite. Since CC1–CC7 yield as theorems all the valid formulas of CC, they yield as a theorem the following valid formula of CC:

$$\alpha^* = -(-(-\alpha^* \cup \alpha^*) \cup -(-\alpha^* \cup \alpha^*)).$$

Hence PC1–PC7 yield as a theorem the following tautology: $\alpha \equiv \sim(\sim(\sim\alpha \vee \alpha) \vee \sim(\sim\alpha \vee \alpha))$, or, for short, $\alpha \equiv (\alpha \equiv \alpha)$. Hence, by virtue of PC8, α is a theorem of PC.[7] That PC1–PC7 and PC9 also yield all tautologies as theorems can be demonstrated as follows. If $\alpha \equiv \alpha$ and $(\alpha \equiv \alpha) \equiv \alpha$ are theorems of PC, then, by virtue of PC9, so is α. But, by virtue of PC4, $\alpha \equiv \alpha$ is a theorem of PC. Hence, if $(\alpha \equiv \alpha) \equiv \alpha$ is a theorem of PC, then so is α. Hence, by about the same reasoning as before, if α is a tautology, then α is a theorem of PC.[8]

II

Of the two extra rules just considered, the first might have appealed to Whitehead less than the second. When PC8 rather than PC9 is thrown in with PC1–PC7, *modus ponens*, the one rule of inference for PC officially acknowledged in *Principia Mathematica*, holds indeed in only one of its two guises. The same sort of thing happens with a good many sets of metaaxioms and rules of inference for PC recently offered in the literature and hence may be worth reviewing in detail.[9]

A finite sequence of formulas of PC is said to be a proof in PC if each formula in the sequence is an axiom of PC or follows from previous formulas in the sequence by application of a (primitive) rule of inference of PC; a formula of PC is next said to be provable in PC or to be a theorem of PC if it is the last formula of a proof in PC; a finite sequence of formulas of PC is next said to be a derivation in PC with n ($n \geq 0$) formulas $\alpha_1, \alpha_2, \ldots$, and α_n of PC as assumption formulas if each formula in the sequence is one of $\alpha_1, \alpha_2, \ldots$, and α_n, or is an axiom of PC, or follows from previous formulas in the sequence by application of a (primitive) rule of inference of PC; a formula of PC is finally said to be derivable in

[7] The argument offered in the text is a simplification of one due to Professor Porte.

[8] For another proof that PC1–PC7 and PC9 yield all tautologies as theorems, see the paper mentioned in footnote 6.

[9] See, for example, H. Hiż, *The Journal of Symbolic Logic*, **24** (1959), 193–202; A. R. Anderson and N. D. Belnap, Jr., *ibid.*, 301–302; and J. Porte, *Comptes rendus des séances de l'Académie des Sciences*, **251** (1960), 188–189.

PC from n ($n \geq 0$) formulas $\alpha_1, \alpha_2, \ldots$, and α_n of PC as assumption formulas if it is the last formula of a derivation in PC with $\alpha_1, \alpha_2, \ldots$, and α_n as assumption formulas.

Writing '$\alpha_1, \alpha_2, \ldots, \alpha_n \vdash \beta$', where $n \geq 0$, for 'β is derivable in PC from $\alpha_1, \alpha_2, \ldots$, and α_n as assumption formulas' and '$\vdash \alpha$' for 'α is a theorem of PC', we are accordingly led to distinguish [10] between two versions of *modus ponens:* the so-called *provability* version, which reads

$$\text{If } \vdash \alpha \text{ and } \vdash \alpha \supset \beta, \text{ then } \vdash \beta$$

and is to be referred to here as MP1, and the so-called *derivability* version, which reads

$$\alpha, \alpha \supset \beta \vdash \beta$$

and is to be referred to here as MP2.

Whether both versions of *modus ponens* are forthcoming as rules of inference of PC depends on the metaaxioms and rules of inference PC is fitted with, as the following should show. To simplify matters I shall presume that the metaaxioms and rules of inference in question are such that:

(a) If α is a theorem of PC, then α is a tautology,

and

(b) If α is a tautology, then α can effectively be shown to be a theorem of PC, that is, instructions can be supplied for constructing a sequence of formulas of PC of which α is the last formula.

Conditions (a) and (b) are met by PC1–PC7 and PC8, by PC1–PC7 and PC9, by Whitehead and Russell's own metaaxioms and rules of inference for PC, and by the recent sets of metaaxioms and rules of inference alluded to at the opening of II.

(1) When, as is the case with *Principia Mathematica*, a rule reading: "From α and $\alpha \supset \beta$ to infer β," appears among the (primitive) rules of inference of PC, both versions of *modus ponens* are of course forthcoming as rules of inference of PC.

(2) In all other cases MP1 is provable as follows as a metatheorem of PC: If two formulas α and $\alpha \supset \beta$ of PC are theorems of PC, then, by condition (a) above, α and $\alpha \supset \beta$ are tautologies; hence, by the definition of a tautology and the truth table for '$\supset$', β is a tautology; and hence, by condition (b) above, β can effectively be shown to be a theorem of PC. The foregoing proof of MP1 is effective.

[10] The same distinction is drawn and studied by Professor Porte in the paper listed in footnote 9.

(3) In the same cases as under (2), MP2 sometimes fails for some pairs of formulas α and β of PC and hence is sometimes not provable, whether effectively or not, as a metatheorem of PC.

(4) MP1 and MP2 qualify as derived rules of inference of PC, that is, as licenses for passing (in the course either of a proof of PC or of a derivation of PC) from two formulas α and $\alpha \supset \beta$ of PC to β itself, when and only when effectively provable as metatheorems of PC. In the same cases as under (2), therefore, MP1 is always forthcoming as a derived rule of inference of PC; MP2, on the other hand, is not always so forthcoming.

It comes as a surprise that PC may be fitted with such metaaxioms and rules of inference that MP2 is not forthcoming as a derived rule of inference of PC. PC1–PC7 and PC8, however, are a case in point, as Professor Jean Porte recently pointed out to me. Consider the two formulas 'p' and '$p \supset q$' of PC, where 'p' and 'q' are two propositional variables of PC. If any formula of PC, 'q', for example, is to be obtained from one or two other formulas of PC by application of PC5, PC6, PC7, or PC8, the one or two other formulas in question must be biconditionals. 'p' and '$p \supset q$', however, are not biconditionals. Hence 'q' cannot be obtained from 'p' and '$p \supset q$' by application of PC5, PC6, PC7, or PC8. MP2 is therefore not provable as a metatheorem of PC when PC1–PC7 and PC8 are elected to serve as metaaxioms and rules of inference for PC.[11]

The situation, however, changes drastically when PC1–PC7 and PC9 are elected to serve in that capacity: MP2 is then effectively provable as a metatheorem of PC, and hence both MP1 and MP2 are forthcoming as derived rules of inference of PC. Proof of MP2 is as follows. $\alpha \equiv (\alpha \equiv (\alpha \equiv \alpha))$ is effectively provable as a theorem of PC by means of PC1–PC7. Hence, by means of PC9, $\alpha \equiv (\alpha \equiv \alpha)$ is derivable in PC from α and $\alpha \supset \beta$ as assumption formulas. But $(\alpha \supset \beta) \equiv (\alpha \equiv (\alpha \,\&\, \beta))$ is effectively provable as a theorem of PC by means of PC1–PC7. Hence, by means of PC9, $\alpha \equiv (\alpha \,\&\, \beta)$; hence, by means of PC9 again, $\alpha \,\&\, \beta$; and hence, by means of PC7, $(\alpha \equiv \alpha) \,\&\, \beta$, are successively derivable in PC from α and $\alpha \supset \beta$ as assumption formulas. But $((\alpha \equiv \alpha) \,\&\, \beta) \equiv \beta$ is effectively provable as a theorem of PC by means of PC1–PC9. Hence, by means of PC9, β is derivable in PC from α and $\alpha \supset \beta$ as assumption formulas. The foregoing proof of MP2 is effective.[12]

In view of the results just arrived at, Whitehead might have preferred PC9 over PC8 as a way of converting postulates for CC into full-fledged postulates for PC.

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[11] The same result holds true with PC10 or PC11 in place of PC8.

[12] The same result holds true with PC12 or PC13 in place of PC9.

WHITEHEAD AND THE PROBLEM OF EXTENSION

IN all periods of his thought the fundamental problems of cosmology were Whitehead's primary concern, and his preoccupation with them resulted in his most original contributions to philosophy. The advance of his thought from the paper "On Mathematical Concepts of the Material World" in 1905 to *The Principles of Natural Knowledge* and *The Concept of Nature* in 1919 and 1920 and on to *Process and Reality* in 1929 has been an advance from a restricted to an ever more comprehensive handling of the issues. The advance was constituted by an increasing penetration into these problems and an increasing realization of which issues are truly basic.

Even in the early paper it was clear to Whitehead that in an important respect the cosmological issues centered on the problem of extension; it was with some aspects of this problem that this paper was concerned. At that stage of his thought Whitehead assumed, along with the whole philosophical tradition, that extension was essentially spatial extension. Even then, however, he realized that the recent advances in scientific theory, culminating in the theory of relativity, had made time crucial. But it was only in his middle period, with his doctrine of "events," that he was able coherently to bring time into the notion of extension: events are essentially spatiotemporal—they have a temporal extensiveness as well as a spatial extensiveness.

The fundamental position of this doctrine is that extensiveness is the extensiveness of events: "Every event extends over other events, and every event is extended over by other events" (CN 59). Extension is an ultimate and intrinsic feature of actuality.

One of the most important aspects of this feature of extension is that it is continuous. As Whitehead wrote, "the continuity of nature arises from the continuity of extension" (CN 59); that is to say, "the continuity of nature is the continuity of events" (CN 76).

This doctrine of events is a formulation, in a philosophical theory, of the unification of space, time, and matter (or "the physical"), achieved by the scientific theory of relativity. The three kinds of real existence of the Newtonian cosmology are, in the doctrine of events, replaced by one real existent, namely, a continuous nature constituted by a continuity of events.

The analogy of Whitehead's position with that of Descartes is noteworthy. Descartes had unified spatial extension and the physical or material as *res extensa;* for Descartes the physical *res* is a

continuous spatial extension. That is, for Descartes, matter and spatial extension are not two reals; the two concepts are integrated as one real existent. Whitehead has gone one step further and analogously integrated the physical with a continuous spatiotemporal extensiveness as one real existent. Einstein has pointed to the analogy of the position of the scientific theory of relativity —especially of the general theory—with that of Descartes.[1] Whitehead's doctrine at this stage was, in respect to this basic ontological issue, close to that of Einstein—despite the important differences between the theories of Whitehead and Einstein. The two are fundamentally in agreement in integrating the physical with space and time into a single real existent and in maintaining continuous extensiveness as an ultimate feature of the real.

After writing *The Principle of Relativity* (1922), however, Whitehead began to see that his doctrine involved some very grave difficulties. A crucial difficulty concerned the relativity of space and time. Whitehead had maintained a relativistic theory; but is it possible consistently and coherently to do so on the basis of a conception of spatiotemporal extension as the ultimate feature of the actual? For does not this imply that spatiotemporal extensiveness is absolute and not relative?

Whitehead came to recognize that the central problem that has to be explicitly faced is the ontological status of extension. Can continuous extension consistently be accepted as a real? That is, can actuality consistently be regarded as continuously extensive? In the few years before the appearance of *Science and the Modern World* (1925) Whitehead subjected this problem to a scrutiny and analysis strikingly analogous to that to which Leibniz had subjected the Cartesian conception of extension. Whitehead's analysis was made in independence of that of Leibniz, and it is thus all the more interesting and significant that the outcome in both cases should be so similar.

The outcome of Whitehead's reflection on this problem is expressed in the following extremely important passage in *Process and Reality:*

> It cannot be too clearly understood that some chief notions of European thought were framed under the influence of a misapprehension, only partially corrected by the scientific progress of the last century. *This mistake consists in the confusion of mere potentiality with actuality. Continuity concerns what is potential; whereas actuality is incurably atomic* (PR 95, my italics).

[1] Cf. Einstein, *Relativity, the Special and General Theory*, tr. by Robert W. Lawson (New York: Hartsdale House, 1947).

The recognition that continuity pertains to potentiality and not to actuality is central to the enormous advance in thought of Whitehead's later period. Herein lies a basic factor making the doctrine of *Process and Reality* importantly different from the doctrine of *The Concept of Nature.* For the doctrine of his middle period still involved that confusion.

In his later period Whitehead argued that "Newton, in his treatment of space, transforms potentiality into actual fact, that is to say, into a creature, instead of a datum for creatures" (PR 123). In other words, "Newton in his description of space and time has confused what is 'real' potentiality with what is actual fact" (PR 113). This is also true of Whitehead's own doctrine in his middle period: that events are the actual existents and that events are essentially continuous. Thus this doctrine ascribes continuity to actuality.

I have said that the outcome of Leibniz's reflections on this problem is strikingly close to that of Whitehead. Leibniz saw very clearly that the basic problem was the ontological status of extension, which is why he subjected the Cartesian and Newtonian doctrines to such careful analysis. His conclusion was that extensive continuity could not be a fundamental feature of the actual, either as itself a kind of actuality (Newton) or as the essential character of the physical (Descartes). On the contrary, like Whitehead later, Leibniz saw that extension involved continuity and that continuity pertained to possibility (or potentiality), not to actuality. He states the point as follows in a letter to De Volder:

> But a continuous quantity is something ideal which pertains to possibles and to actualities only insofar as they are possible. A continuum, that is, involves indeterminate parts, while, on the other hand, there is nothing indefinite in actual things, in which every division is made that can be made. Actual things are compounded as is a number out of unities, ideal things as is a number out of fractions; the parts are actually in the real whole but not in the ideal whole. But we confuse the ideal with real substances when we seek for actual parts in the order of possibilities, and indeterminate parts in the aggregate of actual things, and so entangle ourselves in the labyrinth of the continuum and in contradictions that cannot be explained.[2]

Because continuity pertains to possibility and not to actuality, Leibniz comes to the following conclusion as to the status of spatial and temporal extensiveness, expressed in his third letter to Clarke:

> As for my own opinion, I have said more than once that I hold space to be something merely relative, as time is; that I hold it to be an order of co-

[2] *Leibniz's Philosophical Papers and Letters,* tr. by L. E. Loemker (Chicago: University of Chicago Press, 1956), Letter X, p. 879.

existence as time is an order of successions. For space denotes, *in terms of possibility*, an order of things which exist at the same time.[3]

Space is an order of things coexisting, but not simply that; it is an order "in terms of possibility." As he stated the point in a letter to De Volder:

For space is nothing but the order of existence of things possible at the same time; while time is the order of existence of things possible successively.[4]

Since extensiveness is not a character of actuality, but pertains to possibility, this means, Leibniz held, that the ontological status of extension as possibility must be that of "ideality." Leibniz went one step further, maintaining that "ideality" must be interpreted as "phenomenality." The Kantian doctrine follows Leibniz in this, as Gottfried Martin has shown.[5] And many interpreters of the scientific theory of relativity have, implicitly or explicitly, adopted an essentially Kantian position in regard to the ontological status of extension.

Whitehead explicitly rejects this position. He would agree with Leibniz in maintaining that the ontological status of extension as possibility must be that of "ideality." He differs from Leibniz and the Kantian tradition following Leibniz, however, in regard to the interpretation of "ideality." Whitehead rejects the view that there is no alternative but to interpret "ideality" as "phenomenality." This rejection is a rejection of the whole post-Cartesian tradition of subjectivism based on an ontological dualism, in accordance with which "ideas" are essentially mental existents. Instead Whitehead returns to an Aristotelian conception of "Idea" or "Form." Like Aristotle, he regards "Ideas" or "eternal objects" as the "forms of definiteness" of actuality. In their essential nature forms transcend actuality—which does not mean that they exist apart from actuality. Whitehead's chief difference from Aristotle is in regard to this "transcendence" of form. Form *qua* form is a metaphysical ultimate which constitutes the "possibility" for actuality; eternal objects are "pure potentials for the specific determination of fact" (PR 32). "Possibility" (or "potentiality") is the correlative of "actuality"; it is thus not reducible to "actuality." The factor of possibility in the universe belongs to form. So Whitehead is at one with Leibniz in maintaining that the ontological status of extension as possibility must be

[3] *Ibid.*, p. 1108. My italics.

[4] *Ibid.*, p. 874.

[5] G. Martin, *Kant's Metaphysics and Theory of Science*, tr. by P. G. Lucas (Manchester: University Press, 1955), pp. 38–39.

that of "ideality," but Whitehead's interpretation of "ideality" enables him to develop an important new doctrine.

Leibniz, on his interpretation of "ideality," regards all order as phenomenal. Thus, "in themselves the monads have no situation [*situs*] with respect to each other, that is, no real order which reaches beyond the order of phenomena."[6] This means that Leibniz held a doctrine of *relatio rationalis*. Whitehead, on the other hand, with his different conception of form or ideality, is able to admit real relations. Relations are real because every relation is a relating, an actualization of form.[7] That is, actualities are really (as opposed to merely phenomenally) related by their actualizing a form in common. Any specific structure, pattern, or character displayed by a number of actualities results from their common actualization of a particular form—for structure, pattern, character, are nothing other than form. The actual existence of a structure is the actualization of that structure which, in abstraction from the actualization, is a pure form, and as such a pure possibility. Order is constituted by the exemplification of some structure, some pattern, some character actualized by the actualities in question. Order is an exemplification of some real relatedness between actualities.

Leibniz had seen that extension must be regarded as an order of relatedness between actualities, for extension cannot itself be an actuality. But he conceived the order of relatedness as phenomenal, not real. Descartes, on the other hand, maintained that extension was real by regarding it as itself actual. Whitehead holds, with Leibniz, that extension must be an order of relatedness between actualities and not itself an actuality; but he is able to maintain that the order is a real relatedness. Thus he says:

> We diverge from Descartes by holding that what he has described as primary *attributes* of physical bodies, are really the forms of internal relationships *between* actual occasions, and *within* actual occasions" (PR 471).

The most general "form of relationship" between actualities is the generic form of "extension" or "extensive connection":

> . . . for Descartes the primary *attribute* of physical bodies is *extension;* for the philosophy of organism the primary *relationship* of physical occasions is extensive connection (PR 441).

In the cosmology of *Process and Reality,* therefore, unlike that of *Concept of Nature,* continuous extension is not a basic feature of actuality; actuality as such is not seen as continuously extensive.

[6] Loemker, *op. cit.*, p. 980.

[7] Except, of course, for the relations among forms themselves.

In the earlier doctrine events are continuously extensive; extensive continuity belongs to their essential nature as events. The new doctrine is that "the ultimate metaphysical truth is atomism. The creatures are atomic" (PR 53). That is, actuality is fundamentally and by metaphysical necessity atomic. This does not mean that continuity—and extensiveness—are phenomenal. On the contrary, Whitehead maintains that extensive continuity is real. But its reality is the reality of a relation and not of actuality as such. Also, the character of extensiveness and the character of continuity derive from form and not from actuality. That is to say, for these characters we are to look not to actuality or actualization as such, but to the particular form that is actualized. It is by virtue of the actualization of a particular form that there is extensive continuity. Whitehead acknowledges that "in the present cosmic epoch there is a creation of continuity" (PR 53), but this is not a metaphysical necessity. Rather, he maintains, "extensive continuity is a special condition arising from the society of creatures which constitute our immediate epoch" (PR 53).

Extensiveness as such is a form of relatedness. Considered in itself it is a potentiality for actualities, not an actuality. It is the most general scheme of potential relatedness. That is, it is the most general scheme or structure exhibited by actualities whereby they stand in relationship to one another. It is the structure of potential standpoints.

Whitehead insists, like Leibniz, that extensiveness *qua* extensiveness must be potential, for extensiveness implies continuity, and continuity pertains to potentiality, not to actuality. Thus this potential scheme is an extensive *continuum*. As Whitehead says, as a "continuum [it] is in itself merely the potentiality for division; and actual entity effects this division" (PR 104).

The doctrine Whitehead has put forward in *Process and Reality* is, therefore, that "the real potentialities relative to all standpoints are coordinated as diverse determinations of one extensive continuum. This extensive continuum is one relational complex in which all potential objectifications find their niche. It underlies the whole world, past, present, and future" (PR 103). Thus "this extensive continuum expresses the solidarity of all possible standpoints throughout the whole process of the world" (PR 103). Whitehead stresses repeatedly that the extensive continuum is not to be regarded as actuality, as some kind of "container": "it is not a fact prior to the world; it is the first determination of order —that is, of real potentiality—arising out of the general character of the world" (PR 103). The extensive continuum is the most general form of order.

It would now be requisite to exhibit how space and time are derivative from this generic form of extensive relatedness as specific forms actualized by some, but not necessarily all, actualities. To do so would, however, extend this paper far beyond the assigned limits. Nor is this necessary for our present purpose, which has been to show that in Whitehead's later period he has developed a theory of extension significantly different from that of his earlier period, a theory which constitutes a major contribution to cosmological thought.

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THE PLACE OF MATHEMATICS IN WHITEHEAD'S PHILOSOPHY

LIKE his master, Plato, Whitehead views mathematics as fundamental to philosophy.[1] Indeed, both philosophers have been criticized for an alleged tendency to model philosophy after, if not to identify it with, mathematics. Mere reading should dispel this last misunderstanding. After all, the third level of the divided line is different from the fourth, and Whitehead does explicitly repudiate the method of mathematics in philosophy. Still, even Whitehead's aesthetic-psychological theory of prehension (see PR pt. III)—which seems about as nonmathematical as a philosophical theory could well be—may be viewed in Whitehead's own terms as a temporary makeshift to serve only until symbolic logic has progressed to the point where it can serve as an adequate tool for aesthetics. What then is the relation between Whitehead's philosophy of organism (of which the theory of prehension is one part) and mathematics? One way of approaching an answer to this and similar questions about the place of mathematics in Whitehead's philosophy is to consider Whitehead's conception of the role of pure mathematics in the history of thought. In discussing this question in *Science and the Modern World* (ch. II), Whitehead compares the development of mathematics to the development of language with respect to the degree of analysis of the concepts represented and concludes that the intellectual function of pure mathematics consists in its "resolute attempt to go the whole way in the direction of complete analysis, so as to separate

[1] See, e.g., the last sentences of Whitehead's last book: "Philosophy is akin to poetry. . . . In each case there is reference to form beyond the direct meanings of words. Poetry allies itself to metre, philosophy to mathematic pattern" (MT 237–238).

the elements of mere matter of fact from the purely abstract conditions which they exemplify" (SMW 37). The value of such mathematical analysis, as a general habit of though, is threefold: (1) it focuses attention, by isolating it, on the *direct aesthetic appreciation of the mere matter-of-factness* of any given occasion of experience; (2) it encourages the *abstraction of the particular entities* (e.g., colors, physical objects, physical magnitudes) involved in any given occasion of experience; and (3) it makes possible the *apprehension of the absolutely general conditions* satisfied by the particular entities involved in any given occasion of experience.

Systematic studies of the abstractions mentioned in (2) and (3), respectively, are, it seems clear, precisely what Whitehead means by natural science (founded on sense perception) and pure mathematics (founded on logic). The direct aesthetic appreciation of experience, moreover, requires for its satisfactory completion a movement away from sheer absorption in the experience itself to a reflective discrimination of details,[2] and this movement, I would suggest, then leads quite naturally to the development of aesthetics (in the sense of systematic analysis of the general character of any occasion of experience).[3] The latter discipline seems to be what Whitehead means by "cosmology" (sometimes, "philosophy" or "metaphysics"). Cosmology, as thus understood, would presumably embrace at least one set of explicitly mathematical theories, namely, the canons of proportion which have figured so prominently in the history of architecture, sculpture, painting, and music.[4] In fact, as we shall find below, cosmology in the strict sense includes much, though not all, of pure mathematics, while all

[2] See, e.g., MT 85–86: "The movement of aesthetic enjoyment is in the opposite direction [to that of logic]. We are overwhelmed by the beauty of the building, by the delight of the picture, by the exquisite balance of the sentence. The whole precedes the details.

"We then pass to discrimination. As in a moment, the details force themselves upon us as the reasons for the totality of the effect. In aesthetics, there is a totality disclosing its component parts."

[3] But how can analysis—which deals in abstractions—ever capture the *concreteness* of an occasion of experience? We encounter here the ultimate paradox of philosophy, whose precise task is, in Whitehead's words, "to exhibit the fusion of analysis with actuality" ("Mathematics and the Good," ESP 86). This paradox is bound up with the concept of philosophical, or metaphysical, truth, which is supposed somehow to be completely universal and yet descriptive of reality, necessary but not reducible to tautologies.

[4] Such theories must, of course, never be employed as mere formulas for producing art works—that way lies academicism. The purpose of formulating such theories should be to emphasize the subtle differences that may exist among the various art works embodying a given canon.

pure mathematical propositions have cosmological significance in a somewhat loosened sense. (Cosmology also includes a kind of aesthetic-psychological analysis, mentioned above, which cannot be easily identified with anything that currently passes for mathematics.) For the purpose of subsequent discussion, therefore, I assume that "mathematics" has three senses in Whitehead's philosophy: mathematics as "logic"; mathematics as "natural knowledge"; and mathematics as "cosmology." I illustrate with the example of the proposition: "one and one make two."

Now, first of all, this proposition (or, better, its symbolic equivalent, "$1 + 1 = 2$") is proved in PM (II *110.643). This means that the proposition is deduced according to a few fixed rules of inference (e.g., *modus ponens* and the rule of substitution for variables) from a small number of (hopefully) purely logical axioms.[5] Like the latter, then, "$1 + 1 = 2$" is a purely logical, or necessary, truth. Such truths are about pure abstractions; more precisely, they contain no individual constants, but only individual variables, and, hence, are incapable of referring to specific, concrete entities. (This would, of course, become apparent in the case of the proposition "$1 + 1 = 2$" if one were to substitute for "1", "2", "+", and "=" their meanings as defined in PM.)

When we turn to the physical application of the proposition "one and one make two," the quality of necessity disappears; applied arithmetic is an empirical science:

> . . . it is only by experience that we can know that any definite process of counting will give the true cardinal number of some class of entities. It is perfectly possible to imagine a universe in which any act of counting by a being in it annihilated some members of the class counted during the time and only during the time of its continuance. . . . The most we can do is to assert that a universe in which such things are liable to happen on a large scale is unfitted for the practical application of the theory of cardinal members.[6]

Thus, one and one might *not* always make two if two similar entities, during the process of being counted, coalesced into a single entity indistinguishable from the other two. It is evident that Whitehead is here envisaging a sequence of physical processes

[5] The precise characterization of "purely logical" propositions is, of course, a difficult problem, and Whitehead's criterion—a proposition "true *in virtue of its form*" ("Indication, Classes, Numbers, Validation," *Mind*, **43**, 296)—amounts to little more than a suggestive hint of a solution to the problem. In any case, Whitehead's *ultimate* criterion of what constitutes a logical truth is "self-evidence of pattern" (MT 71); so proofs become simply "tools for the extension of our imperfect self-evidence" (MT 69). Notice the qualifying "imperfect": self-evidence, for Whitehead, is *never* final or incorrigible.

[6] "Mathematics," ESP 202–203.

including the initial grouping of a set of physical objects (e.g., by bringing them into sufficiently close proximity) into a suitable spatial configuration and subsequent acts of identification, memory, and manipulation by the agent who is counting. Each or all of these acts might conceivably influence the physical objects being counted in such a manner as to reduce or augment their original number.

This influence would presumably be physical in character, hence in principle subject to physical laws that would enable one to predict the results of counting in any given case. Two points should be noted, however. (1) The fact that the counting agent himself would be essentially involved in the statement of such physical laws would not necessarily constitute a violation of Whitehead's dictum that natural science excludes all reference to knowing minds, since we may assume that only the counting agent's body would be significant during the counting processes in question. (2) This influence would be in an important sense *external* to the physical objects being counted, since these objects could be said to exhibit their bizarre [7] arithmetical behavior only while subjected to the influence of a counting agent.

According to Whitehead, the passage from natural science to metaphysics involves (among other things) (1) the introduction of knowing minds, and (2) the investigation of the ontological status of the entities of natural science. Furthermore, any genuinely metaphysical proposition must satisfy three criteria.[8] The proposition must (*a*) be meaningful for any *actual occasion* [9] entertaining it, (*b*) be "general" in the sense that its predicate potentially relates any and every set of actual occasions providing the suitable number of subjects for the predicate, and (*c*) have a "uniform" truth value in the sense that its truth value is the same as that of each of the singular propositions derivable from it by specification of a definite set of logical subjects. In order for the proposition "one and one make two" to be accepted as a genuinely metaphysical proposition, the above criteria imply, first of all, that the entities referred to in the proposition must be actual oc-

[7] Bizarre, that is, relative to the standard laws of pure cardinal arithmetic. It is assumed throughout that physical objects conform to these standard arithmetical laws except when they are perturbed by being counted. This assumption is analogous to the assumption of general relativity that empty physical space is Euclidean except in the presence of gravitational fields.

[8] Slightly paraphrased from PR 300.

[9] Actual occasions—the ultimately real ontological entities in Whitehead's metaphysics—are "drops of experience, complex and interdependent" (PR 28).

casions (rather than, say, physical objects) and that the proposition "one actual occasion and another actual occasion make two actual occasions" must be meaningful for any actual occasion capable of entertaining it. Next, of course, one must ask about the truth value of the proposition and of its singular substitution instances. Within the context of Whitehead's metaphysical scheme it is difficult to imagine an exception to the truth of the above arithmetical proposition, since every actual occasion possesses a unique self-identity (a "real, internal constitution" in Locke's phrase, which Whitehead is fond of quoting), and an actual occasion is a changeless entity fixed for all time once it has come into existence. Accordingly, Whitehead suggests that the most obviously metaphysical propositions are the arithmetical theorems.

It is important, however, to realize that, with a different—but, from Whitehead's point of view, inadequate—choice of ultimate metaphysical categories, the proposition "one and one make two" might not turn out to be uniformly true in all instances. If, for example, "enduring objects" (in Whitehead's special sense —see PR 51) were chosen as ultimate ontological entities, then it might very well be possible for two enduring objects that are ordinarily distinct literally to coincide at a particular instant and then resume their independent existence thereafter. This possibility—which would lead to the falsity of the proposition "one and one make two" at the instant in question—depends on the fact that Whitehead's enduring objects are abstractions and hence two distinct (noncontrary) enduring objects are capable of qualifying a single actual occasion (see PR 302). Since the entities of primary significance in both natural science and everyday affairs are generally *not* actual occasions, there could be a world in which arithmetic was metaphysically valid and yet practically useless. Hence Whitehead's conclusion that

> There is no difficulty in imagining a world—i.e. a cosmic epoch—in which arithmetic would be an interesting fanciful topic for dreamers, but useless for practical people engrossed in the business of life (PR 303).

Such a world might be one in which the fluid state of matter predominated, so that, for example, whenever discrete drops of liquid approached one another they would tend to fuse into a single drop, while under other conditions a single drop would tend to shatter into many smaller drops. Thus, arithmetic might be of little avail in describing the behavior of matter in this world. In general, the applicability of arithmetic to a given species of entities requires that some principle of individuation for these entities remains undisturbed during the aggregative processes

characteristic of the species (see MT 124–125). Incidentally, the central role of *process* in interpreting arithmetic carries over, according to Whitehead, to all other branches of mathematics (see below, p. 575.

Before proceeding to further mathematical examples, I shall attempt to characterize schematically and in general terms the place of mathematics in Whitehead's philosophy.

	Mathematics as Logic	Mathematics as Natural Knowledge	Mathematics as Cosmology
Subject matter.....	Pure abstractions [eternal objects]	Physical objects [corpuscular societies]	Ultimate ontological entities [actual occasions]
Kind of truth.....	Necessary	Empirical	Metaphysical
Method of validation	Deduction	Sense perception	Descriptive generalization

Several things should be noted about the above table. The point of view from which it is constructed is that of Whitehead's metaphysics, the philosophy of organism. This is made explicit by the bracketed entries in the "Subject matter" row: *eternal objects* and *actual occasions* are the two ontologically fundamental types of entities in the universe; *corpuscular societies* represent an ontologically derivative type of entity of particular importance in our own cosmic epoch. Also, each actual occasion is analyzable into components called *prehensions* (or feelings) whose data may be either eternal objects (conceptual prehensions) or other actual occasions (physical prehensions). The ontological status of each of the three kinds of mathematics may now be briefly characterized as follows. The concepts and propositions of pure mathematics have the kind of existence associated with all eternal objects; namely, they participate in the primordial nature of God, which means that God perpetually conceives (or conceptually prehends) each eternal object both in its individual uniqueness and in its interrelations with all other eternal objects. The study of the interrelatedness of mathematical eternal objects[10] is precisely that

[10] These are the eternal objects of the "objective" species; there are also eternal objects of the "subjective" species, which include sensa, emotions, and in general what are sometimes classified as subjective psychological states. Now, of course, eternal objects of the subjective species will also be interrelated in various ways (some of which are studied in part III of PR, "The Theory of Prehension"), and one may well ask whether the analysis of such patterns—Whitehead likes to call them "aesthetic" patterns—of interrela-

"intellectual analysis of types of pattern" (ESP 83) governed by "the necessity of abstract logic" (SMW 39) which Whitehead often alludes to in his discussion of the nature of mathematics. It follows that all pure mathematics has metaphysical significance insofar as it discloses part of the primordial nature of God.

Mathematical eternal objects may be conceptually prehended by finite actual occasions either as consciously entertained ideas (e.g., by individual human minds) or as structures exemplified in some actual occasion or some *nexus*[11] of actual occasions (e.g., the cardinal number or the geometrical form of the nexus). Insofar as the propositions of pure mathematics are truly exemplified by *any* actual occasion or *any* logically appropriate nexus of actual occasions, to that extent pure mathematics becomes amenable to metaphysical interpretation and hence belongs to the part of cosmology concerned with the general theory of actual occasions and of nexūs. Furthermore, when we realize that human mathematicians have finite minds and therefore require specially devised symbols to aid them in discovering the implications of the propositions they choose to entertain, then we also realize that the study of mathematical symbolism and of the general character of propositions as such might themselves be considered promising topics for mathematical investigation. And, indeed, Whitehead's *Universal Algebra* is concerned with both these topics, the theme of the book being the comparative study of various algebraic systems—one of them the algebra of symbolic logic—as a means of gaining insights into the general theory of symbolic reasoning and into algebraic symbolism in particular. The subject matter of universal algebra, therefore, belongs to those parts of cosmology which treat of propositions and of symbolism. In his metaphysical writings (e.g., PR and *Symbolism*), Whitehead tends deliberately to ignore mathematical symbolism and the mathematical analysis of propositions, and concentrates instead on developing a set of principles and a

tions is also to be construed as mathematics. The answer would appear to be that the analysis of aesthetic patterns is not *now* mathematical in character (i.e., it is not a "symbolic examination of pattern with the use of real variables"—see "Analysis of Meaning," ESP 99), but that it may well become so in the future.

It should be noted that Whitehead also sometimes uses "aesthetics" in a still broader sense, as when he says that "All order is . . . aesthetic order" (RM 105). Following this usage, one would have to say that all existing mathematics is simply a branch of the one universal and fundamental discipline —"aesthetics"—which has achieved a high degree of perfection in the analysis of space, quantity, order, and related ideas but whose extension to the realm of subjective psychological states remains an urgent desideratum.

[11] A nexus is a grouping of actual occasions whose unity consists in their prehensions of one another.

terminology, largely aesthetic-psychological in character, capable of dealing with symbolism and with propositions [12] in their broadest senses, so that the emphasis is on nonverbal symbols and on propositional properties other than truth value.

Up to this point we have encountered five of Whitehead's eight basic ontological categories: actual occasions, eternal objects, prehensions, nexūs, and propositions. Now, the datum of a prehension (or the content of a feeling) is prehended in a particular way called its *subjective form*—another of the basic ontological categories. Also, there are indefinitely many modes of synthesis of the other types of entities in a single prehension; each such mode is called a *contrast*.[13] An important example of a contrast can occur when a conceptual prehension of an eternal object, derived from a physical prehension whose datum is dominated by that eternal object, has the subjective form of an *adversion;* then, if the two prehensions are integrated (the result being called a *physical purpose*), there will be a strong tendency for the original physical prehension to be reproduced in later actual occasions (see PR 420–422). The constancy of physical purposes accounts for the persistence of the order of nature, including the relative stability of ordinary physical objects. Thus, physical purposes furnish the ontological foundation for physical laws; in fact, one might even say that a physical purpose whose dominating eternal objects belong to the objective species *is* a physical law. Since, however, the transmission of a physical purpose may be distorted or even stopped by an unfavorable environment, it is to be anticipated that physical laws may alter with time. And, indeed, this is the basis of Whitehead's concept of successive cosmic epochs, each of them characterized by some dominant set of natural laws. Within a particular cosmic epoch (such as our own) the regnant natural laws will appear as contingent truths about the universe, but in a broader and longer cosmological perspective these laws may be seen to represent basic metaphysical necessities.[14] This is perhaps the meaning of the following remarks by Whitehead:

> The physical field expresses the unessential uniformities regulating the contingency of appearance. In a fuller consideration of experience they may exhibit themselves as essential; but if we limit ourselves to nature there is no essential reason for the particular nexus of appearance (R 8).

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[12] It is worth remarking that, for Whitehead, *propositions* constitute a new ontological category, different from either eternal objects or actual occasions, a kind of hybrid between the two (see PR 282 ff.).

[13] The eighth ontological category, *multiplicities*, is of no special interest here.

[14] Not that the universe can be ultimately fathomed as rationally determined throughout. Whitehead asserts that his metaphysical scheme requires

No attempt will be made here to discuss the concepts of necessary, empirical, and metaphysical truth as Whitehead uses them; the first two involve many difficult technical problems which demand—and in recent years have received—much attention from specialists, while the crucial problem of metaphysical truth is perhaps simply how to prevent its collapse into either of the other two concepts. Whitehead treats this last problem by means of examples: he defines metaphysical propositions (see above, p. 568) and offers many instances of such propositions in his metaphysical writings (although the clearest instances of metaphysical propositions, it will be recalled, are said to be arithmetical theorems). Another approach to the problem of metaphysical truth—an approach, incidentally, used with considerable success in the philosophical analysis of necessary and empirical truth—is to study the method of validating metaphysical truths. In the opening chapter of *Process and Reality* Whitehead denotes this method as "descriptive (or imaginative or philosophic) generalization" and characterizes it as "the utilization of specific notions, applying to a restricted group of facts, for the divination of the generic notions which apply to all facts" (PR 8).

If each of the three mathematical disciplines—pure mathematics, applied mathematics, and cosmological mathematics—has its own distinctive subject matter and method, there are, of course, also important interrelations among them. The relation between pure mathematics and physics is evident; e.g., Whitehead points out that the birth of modern physics would have been impossible without the prior elaboration by mathematicians of the various abstract ideas associated with the notion of periodicity (SMW 47). The relation between metaphysics and pure mathematics is more subtle. On the one hand, pure mathematics must recognize some metaphysical limits to its degree of generality: "The generality of mathematics is the most complete generality consistent with the community of occasions which constitutes our metaphysical situation" (SMW 38). That is, if we attempt to imagine actual occasions *totally* different from any known to us, then we cannot legitimately assert even that any pure mathematical proposition holds for such actual occasions: "If anything out of relationship, then complete ignorance as to it" (SMW 38). But what, then, of logical truths (such as the axioms of PM)? Are not these necessarily valid even for totally unknown—or unknowable—entities? Whitehead's answer seems to be that pure mathematics and logic

a "principle of concretion (or limitation)" not discoverable by abstract reason alone but responsible for the arbitrary, inexplicable features of reality. This principle is God.

in this regard have exactly the same status;[15] that both are bounded in their scope by our metaphysical horizons; that the validity of both—which is the validity of deductive reasoning—rests ultimately on a single principle, namely, that "every thing must be just its own individual self, with its own individual way of differing from everything else" (SMW 39); and that this principle is the most general aesthetic, or metaphysical, truth involved in the mere fact of interrelated existence disclosed in every actual occasion.[16]

Although pure mathematics is, in the sense just explained, grounded in a metaphysical principle, it does not follow that pure mathematics is not, in general, an independent and autonomous disclipine. It is only when one attempts to understand the ontological status of pure mathematical entities or the relationship of pure mathematics to other kinds of knowledge that one is forced back on metaphysical considerations. In general, the pure mathematician need not be—perhaps had better not be—a metaphysician. Thus, to return to arithmetic for a moment: the attempt in *Principia Mathematica* to exhibit arithmetical truths as logical truths is unsuccessful, Whitehead holds, because it requires essential reference both to the metaphysical notion of types and to the particularities of (natural) history.[17] Hence Whitehead's later attempt to base arithmetic on what he calls "purely logical constructions"[18] (this being, of course, only one of numerous recent attempts to improve on the Frege-Russell, or "logistic," interpretation of mathematics). Nevertheless, the view that arithmetical theorems are merely more or less disguised tautologies—the usual accompaniment of the logistic interpretation of mathematics—is not ultimately satisfactory, since it fails to account for the relevance of

[15] ". . . logic presupposes metaphysics" (MT 146).

[16] The principle appears as "The Category of Objective Identity" in Whitehead's metaphysical system. See PR 39: "This category expresses that each element has one self-consistent function, however complex. Logic is the general analysis of self-consistency." Cf. MT 82–83, where Whitehead says that "there are two types of inconsistency . . . the logical type and the aesthetic type." Here aesthetics is opposed to logic. A few pages later, however, the previous usage—according to which logic is subordinated to aesthetics—reappears in the remark that "when the topic of aesthetics has been sufficiently explored, it is doubtful whether there will be anything left over for discussion" (MT 86). See also footnote 10 above.

[17] A cardinal number is defined as a complete class of equinumerous classes (of objects); hence every change in the number of physical objects (Whitehead's example is the birth of a litter of pigs) will alter the meaning of each cardinal number.

[18] "Indication, Classes, Numbers, Validation," *Mind*, **43**, 171 (July, 1934), 281–297; "Corrigenda," *ibid.*, 172 (October, 1934), 543. The article *without* the corrections is reprinted in ESP.

arithmetical theorems to the actual world, which, as Whitehead conceives it, is a world of ceaseless process. Now, in order to avoid postulating the existence of numbers in the sense of a realm of eternal abstract static entities only accidentally and sporadically related to the actual world, Whitehead holds that arithmetic (and indeed, all mathematics) is essentially concerned with certain "forms of process (or transition)" (see MT 112, 126). Specifically, "The very notion of number refers to the process from the individual units to the compound group" (MT 127). Arithmetical theorems, then, refer to alternative forms of this aggregative process issuing in the same resultant (e.g., "$2 \times 3 = 2 + 4$"). Even a proposition like "$6 = 6$" Whitehead proposes to interpret as more than a bare assertion of identity. Not all mathematical eternal objects, of course, need be forms of *actual* processes (e.g., the ideas of an abstract geometrical system may remain physically unrealized), but every such eternal object—in fact, *all* eternal objects—must be envisaged as "apt for realization" (SMW 229) in the actual occasions whose totality constitutes the world-process. Finally, nothing that has been said is intended to deny the utility or, indeed, the correctness—up to a point—of treating mathematics in abstraction from the world-process:

> It is true that nothing is finally understood until its reference to process has been made evident. And yet, there is the understanding of ideal relationships in abstraction from reference to the passage of brute fact. In the notion of such relationships there is no transition.
>
> For example, throughout mathematics, in one sense, transition does not enter. The interconnections are displayed in their timeless eternity (MT 64).

As a further illustration of the place of mathematics in Whitehead's philosophy, consider now geometry.[19] In the first place, there is the abstract, usually axiomatic, treatment of: Euclidean geometry, the two main types of non-Euclidean geometry (hyperbolic and elliptic), generalized metrical (or Riemannian) geometry, projective geometry, and topology. Whitehead himself wrote at some length about each of these with the exception of the last, which was in the very process of being created as a discipline in its own right during the years when Whitehead was gradually abandoning his work in pure mathematics (roughly, after 1913). Secondly, there is the four-dimensional physical geometry of space-time (whether classical or relativistic), whose elucidation occupied Whitehead from at least 1906 onward, especially in his *Principles of Natural Knowledge*, *The Concept of Nature*, and

[19] For a much fuller discussion of Whitehead's treatment of geometry, see my *Whitehead's Philosophy of Science* (Chicago: University of Chicago Press, 1960), chs. V and VI.

The Principle of Relativity. In these three books, Whitehead argues on the basis of the general character of sense perception that space-time must be uniform. He then chooses Euclidean (more precisely, pseudo-Euclidean) geometry as the framework in which to formulate current (c. 1920) knowledge of the physical world, while admitting that uniformly elliptic or uniformly hyperbolic geometry would be equally possible, if more complicated, choices.[20] Furthermore, Whitehead attempts to derive all geometrical entities (e.g., points, lines, planes, etc.) by his *method of extensive abstraction* from the fundamental data of sense perception, namely, *events* and extensive relations between events. Finally, there is Whitehead's *theory of extension* which attempts to discover the most general "extensional" properties of the universe. These properties—which Whitehead characterizes as "ultimate metaphysical necessities" (PR 441)—are expressed in terms of certain formally defined relations among *regions*, which are themselves interpretable in terms of the basic categories of the philosophy of organism. Study of the formal assumptions about regions suggests that they are topological entities, and this is in accord with Whitehead's view that the ultimate extensional properties of the universe probably include indefinite divisibility and unbounded extension, but not dimensionality, straightness, measurability, shape, or the distinction between space and time. Thus each actual occasion and each nexus of actual occasions must conform in its extensive properties to the general conditions imposed by what Whitehead calls the *extensive continuum* of regions, but not necessarily to any more specialized (e.g., metrical) conditions.

One other mathematical illustration may be briefly mentioned here, namely, the principle of least action. An interesting fact about this principle from the pure mathematical point of view is that it can be formulated in differential form as the Euler-Lagrange equations and that the latter are mathematically equivalent, for a large class of mechanically important cases, to the differential equations expressing Newton's second law of motion. Whitehead makes use of this fact in developing his theory of relativity, which consists of relativistic laws of gravitation and electromagnetism as well as a law of motion of the least-action type (see R ch. IV). Finally, the prevalence of least-action principles in physics can be given a metaphysical explanation within the context of the philosophy of organism (see SMW 155).

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[20] See, for example, R preface, and chs. II and IV.

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